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## Stoppers Energy Harvesting Efficiency Enhancement via Optimal Linear Controller

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**Abstract.** A stoppers solution to enhance energy harvesting imposes a nonlinear behavior to the system and also a chaotic behavior that increase the mechanical energy transferring from excitation source to electric output power. To increase even more energy conversion it was project an Optimal Linear Control to drive the system to a optimize interaction from vibration source to structural system. A piezoelectric film is attached to a cantilever beam and an electric output load is assembled to extract electric energy from vibration sourced. The stoppers are arranged to improve the system efficiency. Using the indirect piezoelectric effect a controller will insert velocity and acceleration to the system and as main result the system increased efficiency 1.18 times considering output voltage rate.

**Key-words.** Energy harvesting, Efficiency, Optimal Linear Control, Stoppers, Impulse

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## 2<sup>1</sup> Introduction

An important promise solution to clean and sustainable energy source are the energy harvesting systems. To harness energy from small sources as fell gradients of temperature, sound, radio frequency and vibration [12] can be accomplished via electrostatic, electromagnetic and piezoelectric means [8] and a piezoelectric based energy harvesting subject to a vibration sourced is the focusing of this research. A piezoelectric material in direct effect behavior converts mechanical energy into electricity and an energy harvesting system can utilizes vibration as source to supply electrical power [6] to a small electronic device without necessity of a physical connection to any other electrical source. Most of actual research of vibration based energy harvesting are focusing in resonant solution [15, 9, 11] were natural frequency is coincident to excitation vibration frequency. Nevertheless ambient vibration is wide band [5] and resonant effect occurs only for a narrow space of time and the efficiency of this solution remains insufficient for his proposes [16]. A non-linear solution proposed by [2] increase efficiency by rising relation from ambient excitation and energy harvesting system response. Although non-linear has appeared as a better solution it is even insufficient to several applications regarding electrical consumption necessities [16]. A possible solution to enhance energy harnessing can be accomplished by controlled system projects that can improve interaction between excitation source and energy harvesting system vibration [14]. An Optimal Linear Control (OLC) is a controller proposed by [7] that can drive a non-linear dynamical system. Applications of OLC to non-linear systems are accomplished by [13] and specifically applications of OLC to enhancement of energy harvesting systems efficiency were studied by [3]. A stoppers energy harvesting system was proposed by [4] as shown in Figure 1. The experiment presented a significant increase of energy due impact effect. The use of impact as a mean to increase energy harvesting efficiency is a novel proposal that must be deeply explored. In this direction the present investigation will propose a control project based on OLC to a non-linear system to optimize the interaction between the vibration source and the harvesting system. The paper is organized in five chapters: energy harvesting system model, controller project, efficiency gain analysis and conclusions plus acknowledgements.

## 2 Energy Harvesting System Model

A stopper energy harvesting system utilizes the impact to enhance mechanical energy transfer from ambient vibration to the transduction system as shown in Figure 1.

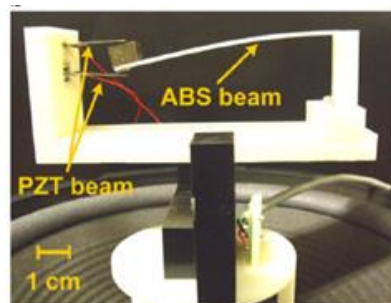


Figure 1: Stoppers energy harvesting system configuration [4].

<sup>3</sup> The mathematical equations for impact energy harvesting system proposed by [4] is:

$$\begin{aligned}
 m\ddot{x} + c_d\dot{x} + k_d x &= ma \sin(\omega_d t) & -d < x < d \\
 m\ddot{x} + (c_d + c_{g1})\dot{x} + k_d x + k_{g1}(x - d) &= ma \sin(\omega_d t) & x \geq d \\
 m\ddot{x} + (c_d + c_{g2})\dot{x} + k_d x + k_{g2}(x + d) &= ma \sin(\omega_d t) & x \leq -d
 \end{aligned} \tag{1}$$

were  $m$  is the proof mass at the tip of the cantilever subject to an exogenous described by  $ma \sin(\omega_d t)$  where  $a$  is the excitation acceleration and  $\omega_d$  is the excitation frequency. The damping ( $c$ ) and the stiffness ( $k$ ) are given according the driving beam ( $c_d, k_d$ ) and the generating beam ( $c_g, k_g$ ). There were two generating beams named beam 1 and beam 2. The gap between the proof mass and the generating beams is  $d$ . The time depending variables is  $x$  for tip cantilever position. According [4] when the displacement is inferior to the gap between the proof mass and the generation beams ( $-d < x < d$ ) there are no electrical generation. Otherwise when the displacement is superior to the gap the generations are given by the amount of displacement of the piezoelectric beam as  $(x - d)$  for upper interference and  $(x + d)$  for lower interference. As a result the impact energy harvesting sourced more mechanical energy to the system and consequently more output power as presented in Figure 2.

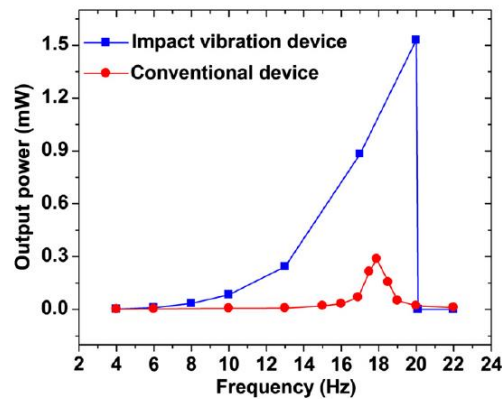


Figure 2: Comparison of impact and conventional energy harvest systems [4].

For numerical simulation it is proposed a comparison of a singular cantilever beam subjected to an exogenous excitation, dimensionless described by [2],

$$\begin{aligned}
 \ddot{x} + 2\zeta\dot{x} + \frac{1}{2}x - \chi v &= f \cos \Omega t \\
 \dot{v} + \Lambda v + k\dot{x} &= 0
 \end{aligned} \tag{2}$$

were  $\zeta$  is damping factor,  $\chi$  is piezoelectric mechanical coupling coefficient,  $\Lambda$  is reciprocal of time constant to capacitive load and  $\kappa$  is piezoelectric electric coupling coefficient. The excitation is described by  $f$  as acceleration rate and  $\Omega$  as frequency rate. The time depending variables are  $x$  for tip cantilever position and  $v$  for output voltage rate. Using a single stopper to produce impact to the system the induced behavior is approximate to a non-ideal power source as described by [10] and adopted as:

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$$\begin{aligned} \ddot{x} + 2\zeta\dot{x} + \frac{1}{2}x - \chi v &= \dot{\varphi}^2 \cos \varphi + \ddot{\varphi} \sin \varphi \\ \ddot{\varphi} &= \ddot{x} \sin \varphi \\ \dot{v} + \Lambda v + \kappa \dot{x} &= 0 \end{aligned} \tag{3}$$

were  $\varphi$  is the phase angle of induced movement by impact force. Isolating  $\ddot{x}$ ,  $\ddot{\varphi}$  and  $\dot{v}$  in Equation 3 and adopting  $y_1 = x$ ,  $y_2 = \dot{x}$ ,  $y_3 = \varphi$ ,  $y_4 = \dot{\varphi}$  and  $y_5 = v$  the space-state of the dynamical system is given as:

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \frac{-2\zeta y_2 - \frac{1}{2}y_1 + \chi y_5 + y_4^2 \cos y_3}{(1 - \sin^2 y_3)} \\ \dot{y}_3 &= y_4 \\ \dot{y}_4 &= \frac{\left(-2\zeta y_2 - \frac{1}{2}y_1 + \chi y_5 + y_4^2 \cos y_3\right) \sin y_3}{(1 - \sin^2 y_3)} \\ \dot{y}_5 &= -\kappa y_2 - \Lambda y_5 \end{aligned} \tag{4}$$

Applying Runge-Kutta fourth order in equation (4) for dimensionless parameters  $\zeta = 0.01$ ,  $\Omega = 0.8$ ,  $\chi = 0.05$ ,  $\kappa = 0.5$ ,  $\Lambda = 0.05$  and  $f = 0.083$  [2] and initial conditions  $x(0) = 1$ ,  $\varphi(0) = 1$ ,  $\dot{x}(0) = 0$ ,  $\dot{\varphi}(0) = 0$  and  $v(0) = 0$  it is determined the voltage rate and the time history considering samples a from 0 to 2,500 in interval of 0.1 totalizing 25,000 time sample according Figure 3a: output voltage and 3b: Time history.

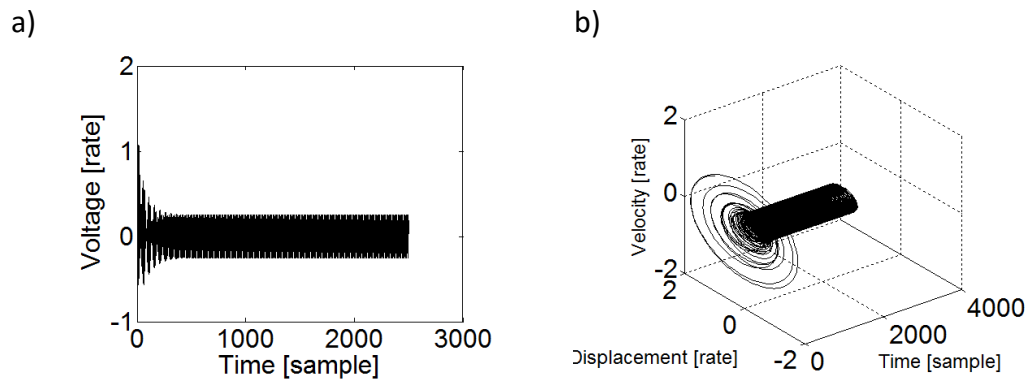


Figure 3: Dynamical behavior of the energy harvesting system.

### 3 Controller Project

An Optimal Linear Control (OLC) is defined to improve system efficiency, which guarantees the linear control application in non-linear systems. Considering a controlled non-linear system given by [1]:

$$\dot{y} = A(t)y + h(y) + Bu, \quad y(0) = y_0 \tag{5}$$

<sup>5</sup> were  $y \in \mathfrak{R}^n$  is a state vector,  $A(t) \in \mathfrak{R}^{n \times n}$  is a boundary conditions matrix (parameters) which elements are time depending,  $B \in \mathfrak{R}^{n \times m}$  is a matrix of constants,  $u \in \mathfrak{R}^m$  is a control vector and  $h(y) \in \mathfrak{R}^n$  is a vector which the elements are continuous non-linear functions,  $h(0) = 0$ . It is highlighted that the chosen of  $A(t)$  is not the only influence of controller efficiency. For a finite time interval and  $A, B, Q \in \mathfrak{R}$  been matrixes of constants elements, the positive defined matrix  $P$  is the solution of the algebraic non-linear Riccati equation, dada given by:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{6}$$

Applying the OLC controller to the defined system the controller produces a gain vector  $G = [0.1346 \quad -0.1046 \quad 0.1000 \quad -0.1356 \quad -0.0082]$  and consequently applying the equation  $A_c = A - BG$ , were  $A$  is the original state matrix  $A_c$  is the controlled state matrix,  $B$  is the actuation vector and  $G$  is the result gain vector, it is possible to determine the controlled system parameters as shown in Table 1. The controlled system is also stable as all eigenvalues do have real part negative.

Table 1: Controlled system parameters.

$\zeta$	1.0000e-04
$\chi$	0.0582
$\Lambda$	0.0418
$\kappa$	0.3954

Applying Runge-Kutta forth order in equation (4) for dimensionless parameters  $\zeta = 0.01$ ,  $\Omega = 0.8$ ,  $\chi = 0.05$ ,  $\kappa = 0.5$ ,  $\Lambda = 0.05$  and  $f = 0.083$  [2] for the system without control and the parameters given in Table 1 for controlled parameters and initial conditions  $x_1(0) = 1$ ,  $\varphi(0) = 1$ ,  $\dot{x}_1(0) = 0$ ,  $\dot{\varphi}(0) = 0$  and  $v(0) = 0$  it is determined the rate of voltage and time history considering samples a from 0 to 2,500 in interval of 0.1 totalizing 25,000 time sample according Figure 3a: output voltage and 3b: Time history

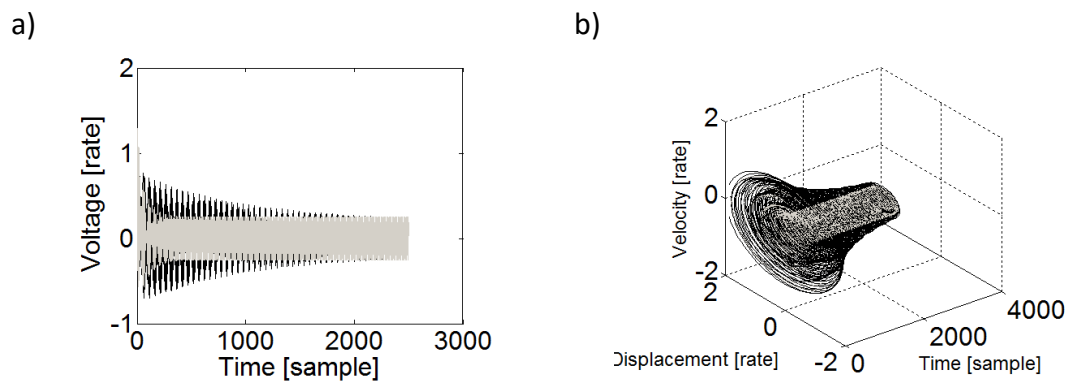


Figure 3: Efficiency gain via active control.

## <sup>6</sup> 4 Efficiency Gain Analysis

As main result the controller increased the system efficiency in convert mechanical energy to electric output voltage. The Table 2 has shown the gain of efficiency via OLC active control according root mean square output voltage value.

Table 2: RMS voltage rate.

Without control	0.1927
OLC with LQR	0.2292
Controller Gain	1.1894

## 5 Conclusions

As main conclusion the impact energy harvesting presented a feedback parameter that converts the system behavior from linear to nonlinear because a direction of movement of the driving beam to the generation beam. Is also highlighted that is possible to approximate the impacted system model to a non-ideal power source system.

As main conclusion applying OLC control to the system results an increase of 1.1894 times of output voltage from the system without control compared to the controlled system.

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