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Resonant Orbital Dynamics of CBERS Satellites

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Abstract.Synchronous satellites have been studied in literature, due to the research of resonant orbits. In this work, CBERS - 2 (China-Brazil Earth Resource Satellite) satellite is investigated observing resonance effects which compose your orbit. Energy's curves are observed indicating the presence of Kozai's resonance in your orbit.

Keyword. CBERS Satellites, Orbital Motion, Resonance

1 Introduction

The technological development has provided several space missions with different goals and proposals. The artificial satellites orbiting the Earth represents 29 % of the cataloged objects, but, only 7 % are operational spacecraft. Considering approximately 10000 cataloged objects around the Earth, one can verify the distribution of the others objects as: 41 % of miscellaneous fragments, 13 % of mission-related objects and about 17 % of rocket bodies. The uncatalogued objects larger than 1 cm are estimated in some value between 50000 and 600000 [5].

Currently, the orbital motions of the cataloged objects can be analyzed using the 2-Line Elements set of the NORAD (North American Defense) [9]. The TLE are composed by seven parameters and epoch. These data can be compared, for example, with the model of the orbit propagator on board in the artificial satellite. A similar study is done for the Brazilian satellite CBERS-1 in cooperation with China. The CBERS satellites provided important scientific advances to Brazil and the images generated are used in several areas, as water resources monitoring, urban growth, deforestation control, soil occupation and education [1].

The objects in the space between the Earth and the Moon are classified in low earth orbit (LEO), medium earth orbit (MEO) and geostationary orbit (GEO). In the last years,

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the LEO region have been studied about the space debris mitigation due to the increasing number of this kind of object through the years. These aspects englobe the observation, spacecraft protection and collision avoidance [2].

The present distribution of objects by the value of the mean motion n indicates the commensurability between the frequencies of the mean motion of the object and the Earths rotation motion. It is verified that most of objects are in the region $13 \leq n(rev/day) \leq 15$.

The space between the Earth and the Moon has several artificial satellites and distinct objects in some resonance. Synchronous satellites in circular or elliptical orbits have been extensively studied in literature, due to the research of resonant orbits characterizing the dynamics of these satellites (see [6–8] and references here in)

In this work, resonant orbital motions of the CBERS satellites are studied using the TLE files of the NORAD. Figures show the time behavior of the orbital keplerian elements, resonant angles and resonant periods. Energy's curves are observed in the (ω, e) plane of the orbital motions indicating the presence of Kozai's resonance in their orbits.

2 Purpose and Methodology

In this section, the TLE data are used to verify objects in resonant orbital motions, specifically space debris and CBERS satellites around the 14:1 resonance [9].

To study the resonant objects using the TLE data, a criterium is established for the resonant period $Pres$, by the condition $Pres > 100$ days. Note that, the resonant period is related with a resonant angle which can influence the orbital motion of a particular object, a CBERS satellite or space debris, for example. The value of $Pres$ helps to understand the influence of each resonant angle and a minimum value is established for the resonant period. $Pres$ is calculated by the relation,

$$Pres = \frac{2\pi}{\dot{\phi}_{lmpq}}, \tag{1}$$

and $\dot{\phi}_{lmpq}$ is obtained from [4],

$$\phi_{lmpq}(M, \omega, \Omega, \Theta) = (l - 2p + q)M + (l - 2p)\omega + m(\Omega - \Theta - \lambda_{lm}) \ , \tag{2}$$

where a, e, I, Ω, ω, M are the classical keplerian elements: a is the semi-major axis, e is the eccentricity, I is the inclination of the orbit plane with the equator, ω is the argument of pericentre, Ω is the longitude of the ascending node and M is the mean anomaly, respectively; λ_{lm} is the corresponding reference longitude along the equator and Θ is the Greenwich sidereal time. So, $\dot{\phi}_{lmpq}$ is defined as

$$\dot{\phi}_{lmpq} = (l - 2p + q)\dot{M} + (l - 2p)\dot{\omega} + m(\dot{\Omega} - \dot{\Theta}) \ . \tag{3}$$

Substituting $k = l - 2p$ in (3), one finds

$$\dot{\phi}_{kmq} = (k+q)\dot{M} + k\dot{\omega} + m(\dot{\Omega} - \dot{\Theta}) \ . \tag{4}$$

The terms $\dot{\omega}$, $\dot{\Omega}$ and $\dot{M}$ can be written as [2].

$$\dot{\omega} = -\frac{3}{4}J_2 n_o \Big(\frac{a_e}{a_o}\Big)^2 \frac{(1-5cos^2(I))}{(1-e^2)^2},$$

$$\dot{\Omega} = -\frac{3}{2}J_2 n_o \Big(\frac{a_e}{a_o}\Big)^2 \frac{(cos(I))}{(1-e^2)^2},$$

$$\dot{M} = n_o - \frac{3}{4}J_2 n_o \Big(\frac{a_e}{a_o}\Big)^2 \frac{(1-3cos^2(I))}{(1-e^2)^{3/2}}. \tag{5}$$

a_e is the Earth mean equatorial radius, a_e=6378.140 km, J_2 is the second zonal harmonic, $J_2 = 1,0826 \times 10^{-3}$.

The term $\dot{\Theta}$ in rad/day is

$$\dot{\Theta} \approx 1.00273790926 \times 2\pi. \tag{6}$$

In order to use orbital elements compatible with the way in which Two-Line Elements were generated, some corrections are done in the mean motion of the TLE data. Considering as n_1 the mean motion of the 2-line, the semi-major axis a_1 is calculated [2],

$$a_1 = \left(\frac{\sqrt{\mu}}{n_1}\right)^{2/3}, \tag{7}$$

where μ is the Earth gravitational parameter, μ=3.986009 x 10^{14} m^3/s^2. Using a_1, the parameter δ_1 is calculated by the Eq. (8) [2],

$$\delta_1 = \frac{3}{4}J_2 \frac{a_e^2}{a_1^2} \frac{(3cos^2(I)-1)}{(1-e^2)^{3/2}}. \tag{8}$$

Now, the new semi-major axis a_o used in the calculations of the resonant period is defined using δ_1 from the Eq. (8) [2],

$$a_o = a_1 \left[1 - \frac{1}{3}\delta_1 - \delta_1^2 - \frac{134}{81}\delta_1^3\right], \tag{9}$$

and the new mean motion n_o used in the calculations is found considering the semi-major axis corrected a_o,

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$$n_o = \sqrt{\frac{\mu}{a_o^3}}. \tag{10}$$

The simulation identified CBERS satellites and space debris with resonant period greater than 100 days. Several values of the coefficients, k, q and m are considered in the Eq. (2) producing different resonant angles to be analyzed by the Eq. (1).

These studies allow to investigate the real influence of the resonance effect in the orbital dynamics of the CBERS satellites and space debris. The number of resonant objects in comparison with the total number of objects in the TLE data shows the great influence of the commensurability between the mean motion of the object and the Earth's rotation angular velocity on its orbits.

In the next section, the orbital motion of CBERS-2 satellite is studied.

3 Results and Comments

In this section, the real data of CBERS-2 is used to study the possible regular or irregular orbital motion.

Figures 1 show the time behavior of the classical keplerian elements of CBERS-2 satellites. The orbital motion of CBERS-2 satellite satisfies the condition $P_{res} > 100$ days. Figure 2 show the resonant periods and resonant angles.

Observing the time behavior of the orbital elements of the object CBERS-2 in Fig. 1, one can verify possible regular and irregular motions in the trajectories of these objects. The time behavior of the semi-major axis and eccentricity of the CBERS-2 show irregularities. Note that in the interval between 500 and 600 days, Fig. 1, a fast increase in the semi-major axis occurs and these variations is about 300 meters and it may be related with some disturbance added to the motion.

Analyzing the time behavior of the resonant period in Fig. 2 a), it is verified that the resonant angles remain confined for a few days. The term confined means that the orbital motion is inside a region delimited for resonant angles with biggest resonant periods.

To continue the analysis about the irregular orbital motions, the time behavior of the $\dot{\phi}_{kmq}$ is studied verifying if different resonant angles describe the orbital dynamics of these objects at the same moment.

Analyzing the time behavior of the resonant angles in Fig. 2 b), one can verify that all resonant angles have the same α, $\alpha = 3/43$, in different combinations for $(k + q)$. Object CBERS-2 have your orbital motion influenced by resonant angles in the neighborhood of the exact 14:1 resonance and they need a full system with different resonant angles which compose their motions.

To verify the Lidov-Kozai s mechanism in a specified orbital motion, curves of same energy in the (ω, e) plane is verified showing libration and circulation curves. Where ω is the argument of pericentre and e is the eccentricity. The study of the Lidov-Kozais mechanism is based on the parameter h, related with the z component of the angular momentum. The parameter h is given by [3]:

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$$h = (1 - e^2) cos^2(I) = const. \tag{11}$$

where I is the inclination of the orbit plane with the equator.

Figure 3 shows the (ω, e) plane using the TLE data of the CBERS-2 satellite. Figure 3 a) shows circulation and libration regions in the (ω, e) plane for CBERS-2 in the period January/2011 to March/2013. Figure 3 b) shows circulation and libration regions considering the orbital motion of CBERS-2 in the period October/2003 to March/2013, analyzing real data since launch.

This analysis helps to find stable regions in the orbital motions around the Earth.

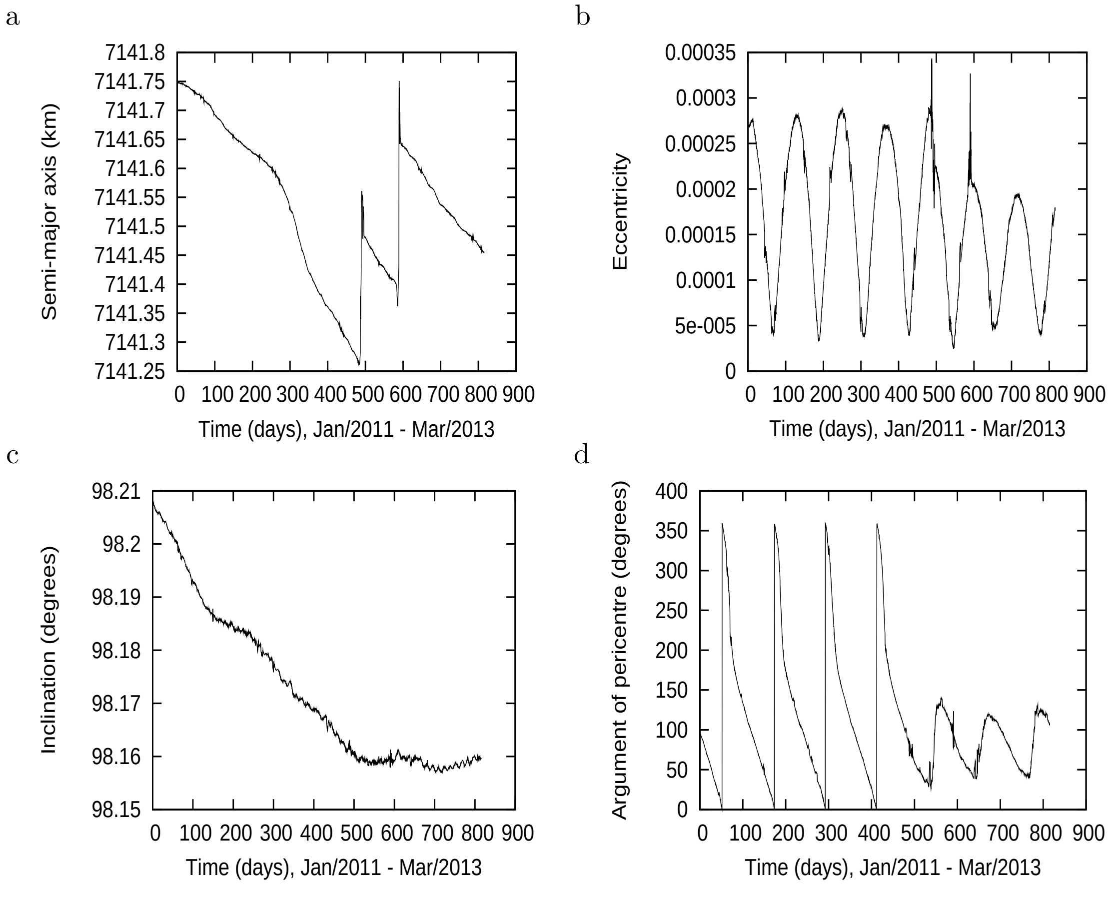

Figura 1: Orbital motion of CBERS-2 corresponding to January/2011 to March/2013; a) Time behavior of the semi-major axis, b) Time behavior of the eccentricity, c) Time behavior of the inclination and d) Time behavior of the argument of pericentre.

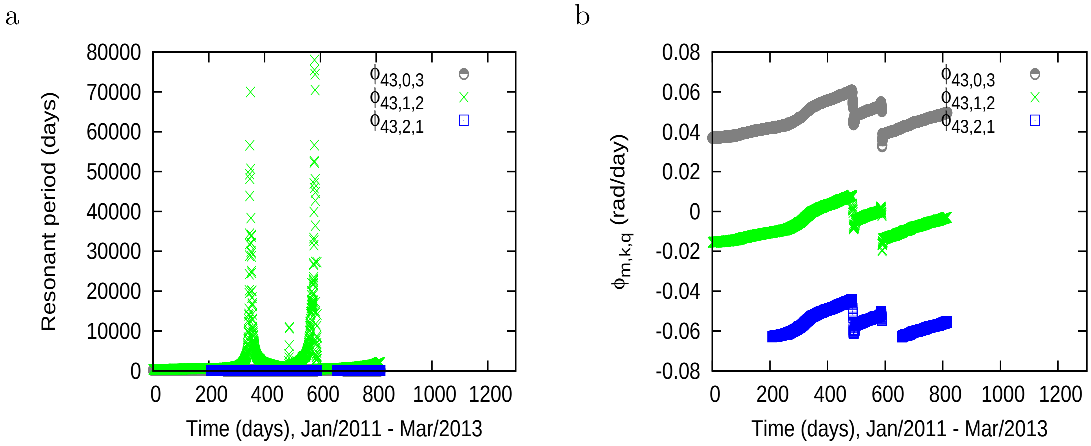

Figura 2: Orbital motion of CBERS-2 corresponding to January/2011 to March/2013; a) Time behavior of the resonant period, b) Time behavior of the $\dot{\phi}_{kmq}$.

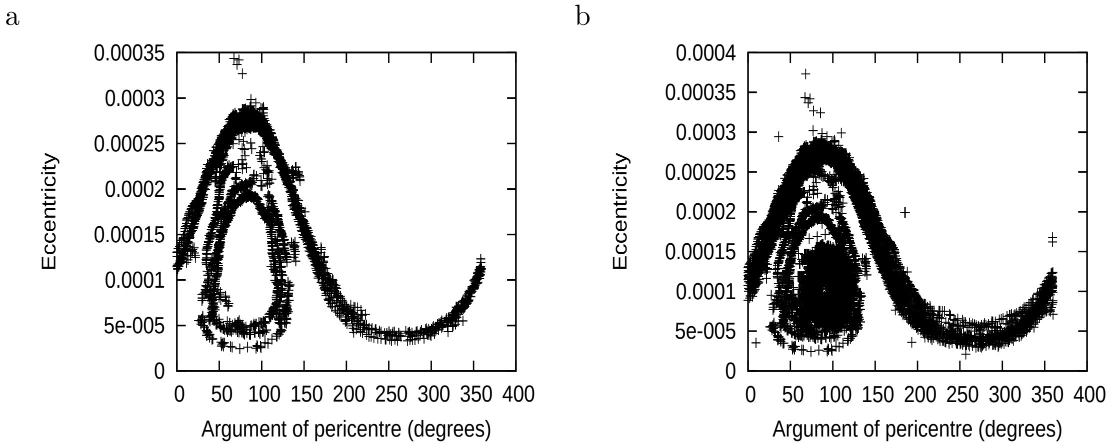

Figura 3: Orbital motion of CBERS-2 satellite. a) Argument of pericentre versus eccentricity corresponding to January/2011 to March/2013; b) Argument of pericentre versus eccentricity corresponding to October/2003 to March/2013.

4 Conclusions

In this work, resonant orbital motions of CBERS-2 (China-Brazil Earth Resource Satellite) satellite is studied using an analytical model and TLE files of the NORAD.

The orbital motions of the CBERS satellites can be corrected during your lifetime, because some disturbances, resonance effects or collision risk can affect their missions. These corrections can be seen by the abrupt change in the values of the semi-major axis. In this way, the study of the resonant angles using real data of the artificial satellites is limited to the period without corrections. However, the study involving space debris

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allows to use a long time and consequently a better analysis about the resonant period in a given region.

The results and discussions show the complexity in the orbital dynamics of this object caused by the resonance effects. Figures show time behavior of classical keplerian elements, resonant angles and resonant periods.

Energys curves are observed in the $(\omega; e)$ plane of the orbital motion of CBERS-2 satellite indicating the presence of Kozais resonance in your orbit.

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