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Nonlinear Dynamic Analysis of a Submerged Hose Using Co-rotational Formulation

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Abstract. Flexible hoses are used in several engineering operations, specially in the offshore industry, they are used to transfer liquid products between FPSO's and also are present in some riser configurations. These hoses undergoes large deformations with small strains, to model this behavior the corotational formulation is implemented along the Newmark integration operator with Newton-Raphson iterations to obtain the nonlinear dynamic equilibrium response. The analysis is divided in two stages; static analysis, to obtain the static configuration when dead loads are applied; and dynamic analysis to obtain the response of the system to time-dependent loads. This two stages are coupled and the dynamic response of a hose under time-varying displacement is simulated.

Keywords. Co-rotational, nonlinear dynamics, large deformation, prestresses, risers.

1 Introduction

Flexible hose are use widely used in the industry, specially in the oil and gas industries. As is well known, in this industries security and fitness for service is of ultimate importance. Flexible hoses are used in different parts of the offshore operations, they are used to transfer products between vessels and also are used in some deep-water riser configurations, like in the hybrid tower configuration. Dynamic analysis of this components is of main importance in virtue of the calculation of the fatigue resistance. Flexible hoses presents large deflections keeping small strains, this geometrical nonlinearity can be modeled using different methods like Total Lagrangian or Updated Lagrangian, both needing the use of stress and strain tensors, another option, the Corotational Formulation, uses the classical beam theory in a local sense and transforms to a global reference. Corotational formalution has been successfully applied in [5], [11] and [9]. The dead and live loads acting on the hose are applied in two different analysis, static configuration is obtained by solving the equilibrium equation using Newton-Raphson method, this static configurations serves as base to the dynamic response analysis, the pre-stress effects of the

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prior static analysis is considered in the global analysis by transferring the internal force vector and the deformation vector.

2 Mathematical Modeling

The Finite Element Method is used in this work to solve the boundary value problem associated to the risers elastic problem. It is assumed the riser undergoes large transverse displacements and rotations with strains remaining small, based on this assumptions the corotational formulation is implemented to solve the geometrically nonlinear problem. The corotational formulation is based on the Total Lagrangian formulation, used extensively in several commercial finite element softwares.

2.1 Corotational Formulation

In the 60's and 70's important authors like [1] and [10] started using corotational orthogonal frames to analise geometrical nonlinearities, [2] used convected frames. Corotational formulations uses a element-fixed orthogonal frame which remains orthogonal as the element deforms elastically. As the element size becomes small enough linear relations are permited locally.

In this formulation three configurations are of main importance, those are: initial configuration, corotational configuration and current configuration. Between the initial and corotational configurations there is only rigid body motion and between the corotational and current configuration pure deformational motion is expected. Axial and angular deformations in the current configuration are measured with respect to the initial configuration, as in the Total Lagrangian formulation. Two coordinate systems should be used: A global inertial framework and a local coordinate system, this coordinate systems undergoes large rotations and displacements as is attached to the riser finite element nodes. The nodal displacements measured from the local system in the current configuration are small enough to allow the use of the linear beam theory, in particular, the Euler-Bernoulli beam theory.

As stated above, the global displacement vector $\mathbf{p}$ is decomposed in two parts: rigid body displacements and deformational displacements, the latter containing elongations and deformational rotations. In two-dimensional beam analysis the extraction of the deformational displacement is made using the local coordinate system rotations and translations. The global and local displacement vectors, $\mathbf{p}$ and $\mathbf{p}_l$ respectively, are shown in equations 1 and 2

$$\mathbf{p} = \left\{ u_1 \quad v_1 \quad \theta_1 \quad u_2 \quad v_2 \quad \theta_2 \right\} \tag{1}$$

$$\mathbf{p}_l = \left\{ u_l \quad \theta_{1l} \quad \theta_{2l} \right\} \tag{2}$$

in equation 2, u_l represents the riser element elongation, θ_{1l} and θ_{2l} represents the angular deformations of the element, see Figure 1. Based on Figure 1 the local and global nodal displacements are related by the following equations

$$u_l = \sqrt{((X_2 + u_2) - (X_1 + u_1))^2 + ((Y_2 + v_2) - (Y_1 + v_1))} - L_0 \tag{3}$$

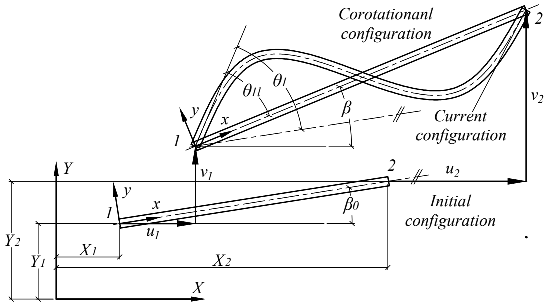

Figure 1: Global and local frames used in the corotational beam element.

$$\beta = \arctan\left(\frac{Y_2 + v_2 - (Y_1 + v_1)}{X_2 + u_2 - (X_1 + u_1)}\right) \tag{4}$$

$$\theta_{il} = \theta_i + \beta_0 - \beta, \quad i = 1, 2. \tag{5}$$

where β represents the rigid body rotation of the local coordinate system, β_0 is the element inclination in the initial configuration.

Following the corotational formulation considerations linear relations for stress-strain from classical beam theory can be used at a local level, the internal axial force and bending moments are obtained as given in 6 and 7

$$N = \frac{EAu_l}{L_0} \tag{6}$$

$$\begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix} = \frac{2EI}{L_0}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{Bmatrix} \theta_{1l} \\ \theta_{2l} \end{Bmatrix} \tag{7}$$

As shown in the work of Crisfield (1991) a transformation matrix $\mathbf{B}$ that transforms the internal force vector expressed in local coordinates, $\mathbf{q}_l$, to global coordinates, $\mathbf{q}$, can be obtained using simple geometrical procedures. The tangent stiffness matrix containing the linear and nonlinear terms is obtained by taking the first variation of the internal force vector as shown in equation 10.

$$\mathbf{B} = \begin{bmatrix} -\cos\beta & -\sin\beta & 0 & \cos\beta & \sin\beta & 0 \\ -\sin\beta/L & \cos\beta/L & 1 & \sin\beta/L & -\cos\beta/L & 0 \\ -\sin\beta/L & \cos\beta/L & 0 & \sin\beta/L & -\cos\beta/L & 1 \end{bmatrix} \tag{8}$$

$$\mathbf{q} = \mathbf{B}^T\mathbf{q}_l, \qquad \mathbf{q}_l = \begin{bmatrix} N & M_1 & M_2 \end{bmatrix}^T \tag{9}$$

$$\delta\mathbf{q} = \delta\mathbf{B}^T\mathbf{q}_l + \mathbf{B}^T\delta\mathbf{q}_l = \mathbf{k}_{tl}\delta\mathbf{p} + \mathbf{k}_{t\sigma}\delta\mathbf{p} = (\mathbf{k}_{tl} + \mathbf{k}_{t\sigma})\,\delta\mathbf{p}. \tag{10}$$

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2.2 External Force Vector

The distributed self-weigth and buoyancy forces, q, in units of force per length, are lumped on the nodes using hermitian functions, this nodal forces always points vertically downwards. This forces are considered as dead loads and are applied incrementally in the static stage of the solution.

$$q = g(\rho_p\, A_p - \rho_e\, A_e) \tag{11}$$

in the former equation g represents gravity, ρ_p and ρ_e are the pipe density and sea water density respectively, A_p and A_e represents the cross-sectional area of the pipe and the outer area obtained using the external pipe diameter.

Time-varying prescribed displacement applied at the right end is modeled as harmonic function of time with known amplitude, frequency and phase. This displacement $u_p(t)$ is imposed using the Penalty Method, physical interpretation of this method requires the addition of a spring of large stiffness, k_p, in the vertical degree of freedom of the right end node and a external force of magnitude $k_p\, u_p$ applied in the same degree of freedom. The imposed displacement $u_p(t)$ is modeled as shown in the next equation

$$u_p(t) = A \sin(\omega\, t + \phi), \tag{12}$$

in this equation A stands for the amplitude of the oscillation, ω represents the frequency of oscillation and ϕ the phase. The value of penalization parameter k_p should be big enough to obtain a pseudo numering decoupling of the elastic system [6].

2.3 Numerical Methods

The equation that governs the dynamic response of the flexible hose, considering the prestress effects due to the dead loading, is composed of the following terms: inertia, energy dissipation (damping) and the internal force. These terms should be in equilibrium with the external time-varying total force, as shown in Equation 13

$$\mathbf{M}\,\ddot{\mathbf{U}} + \mathbf{C}\,\dot{\mathbf{U}} + \mathbf{F}_S^{\text{int}}(\mathbf{U}_S) = \mathbf{F}_S^{\text{ext}} + \mathbf{F}_D^{\text{ext}} \tag{13}$$

$$\mathbf{K}_T\,\mathbf{U}_S = \mathbf{F}_S^{\text{ext}} \tag{14}$$

Note that in Equation 13 the term $\mathbf{F}_S^{\text{int}}(\mathbf{U}_S)$ represents the internal force vector associated to the vector of nodal displacement $\mathbf{U}_S$ that represents the static configuration obtained in the prior static analysis by solving the nonlinear equation 14. The NEWTON-RAPHSON incremental iterative method is applied using the tangent stiffness $\mathbf{K}_T$ presented in equation 10. The vector $\mathbf{F}_S^{\text{ext}}$ contains static nodal forces and $\mathbf{F}_D^{\text{ext}}$ dynamic nodal forces. Assembled mass matrix $\mathbf{M}$ and damping matrix $\mathbf{C}$ are calculated as shown in [3]. Dynamic nodal displacement vector $\mathbf{U}$ is calculated at each time step using the implicit integration method of NEWMARK.

Figure 2 shows the analysis procedure, the left part shows the static analysis and the right part the dynamic analysis. The link between these two analysis is the nodal displacement vector $\mathbf{U}_S$, resulting from the static analysis, based on this vector the internal force vector $\mathbf{F}_S^{\text{ext}}$ is constructed, having this vector, along the mass and damping matrices, the initial nodal acceleration vector is calculated.

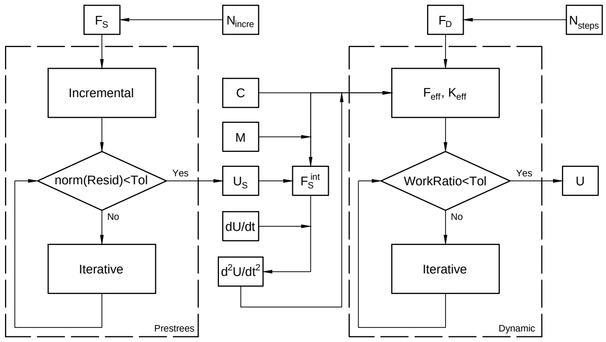

Figure 2: Solution procedure to calculate dynamic response of prestressed hose.

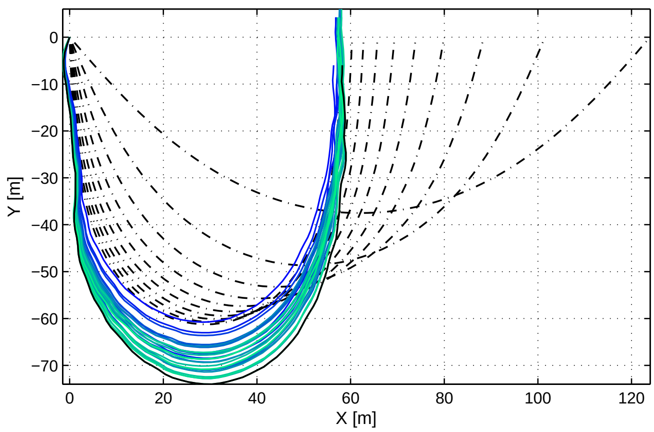

Figure 3: Deformation of the flexible hose at diferent time intervals.

3 Dynamic Analysis

The dynamic response of a submerged flexible hose connecting two floating platforms is going to be analyzed, as shown in Figure 3 the left end allows rotation but no displacement, the vertical displacement of the right end is prescribed. Is supposed that the hose undergoes large deflections and rotations, keeping the strains small enough to consider elastic response. The whole analysis is divided in two stages, each one with different objectives. The first stage objective is to reach the static configuration of the hose under dead loads, as buoyancy and self-weight, in the second stage the dynamic response of the hose under the effects of a time-varying displacement at the right end is obtained.

Table 1 shows the physical properties of the hose and the characteristics of the harmonic displacement. Figure 4a shows the displacement history applied at the right end node. Figures 4b to 4e shows the response of the hose for different points of interest. The static response is not taking in consideration in these figures and only dynamic response is shown.

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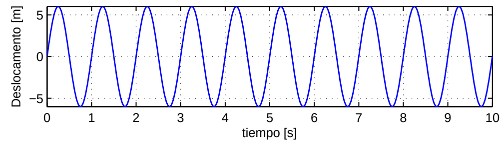
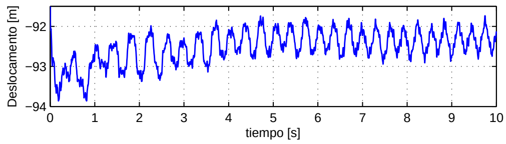
Table 1: Simulation physical parameters.

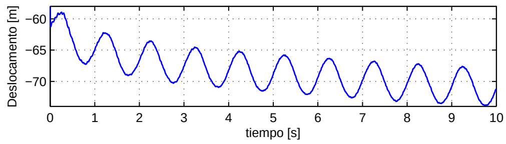
Parameter	Value
External diameter D_e (mm)	273
Wall thickness t_w (mm)	12,7
Total length L (m)	150
Hose density ρ_r (kg/m^3)	7850
Elastic modulus E (Pa)	$2,08 \times 10^{11}$
Harmonic force amplitude A_Y (mm)	8
Harmonic force frequency f (Hz)	1



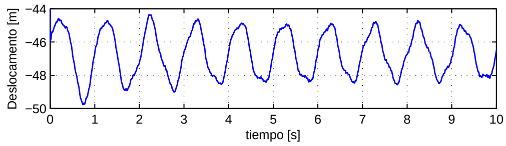
(a) Right end vertical displacement.



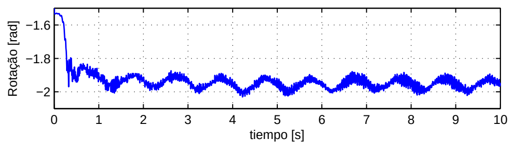
(b) Right end Horizontal displacement.



(c) Middle point vertical displacement.



(d) Middle point horizontal displacement.



(e) Left end angular movement.

Figure 4: Dynamic response for several points on the hose.

4 Conclusions

In two-dimensional analysis, the co-rotational formulation gives a simple way to calculate the tangent stiffness matrix necessary to the calculation of the static and dynamic

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response. This is possible because rotations can be added, in three-dimensional analysis this gets more complex, and geometric methods to determine the relation between infinitesimal displacements in local coordinates and global coordinates must be used. The penalty method used to impose the harmonic displacement introduces numerical error that can destabilize the algorithm, stiffness proportional damping dissipates this errors stabilizing the algorithm.

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