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Identification of nonlinear mechanical systems: Comparison between Harmonic Probing and Volterra-Kautz methods.

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Abstract. This paper proposes a comparison of the performance of two approaches for nonlinear system identification using Volterra series. The first approach uses the harmonic probing method. The second one requires only the input and output data. To illustrate the results and applicability a Duffing system is used.

Palavras-chave. Nonlinear Identification, Volterra-Kautz Series, Harmonic Probing Algorithm.

1 Introduction

The identification of nonlinear behavior in mechanical systems has been extensively studied in recent problems of structural dynamics [10]. Over the past few decades, a wide variety of complex phenomena have encouraged the development of new methods to detect nonlinear effects inherent in complex systems [2, 5]. Among them, the Volterra series have demonstrated to be useful tool and widely used in several engineering applications [6]. The present paper explores two current methods based on Volterra series for nonlinear system identification. The first approach is a white box modeling and the solution is computed by the harmonic probing algorithm using the Fourier transform of the Votlerra kernels known as higher-order frequency response functions (HOFRFs) [1,3,9]. The second one uses the discrete-time Volterra series expanded onto orthonormal Kautz basis [4, 7, 8]. Thus, the goal of the present paper is to evaluate and to compare both methods seeking further applications of structural dynamics. The paper is organized as follows. In section 2, the Volterra series and the relevant aspects of the harmonic probing method are summarized. In section 3, the grey box Volterra-Kautz models are briefly reviewed. A numerical application involving a Duffing oscillator is used to illustrate the methods. Section 4 discusses the results reached by each method. Finally, the final remarks are presented in section 5.

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2 Volterra series and harmonic probing algorithm

A Duffing oscillator with nonlinear effects is described by:

$$m\ddot{y}(t) + c\dot{y}(t) + k_1 y(t) + k_3 y^3(t) = u(t) \tag{1}$$

The Volterra series allows to describe the output $y(t)$ of the system in Eq. (1) as [6]:

$$y(t) = \underbrace{\int_{-\infty}^{\infty} h_1(\tau_1)u(t-\tau_1)d\tau_1}_{y_1(t)} + \underbrace{\iiint_{-\infty}^{\infty} h_3(\tau_1,\tau_2,\tau_3)u(t-\tau_1)u(t-\tau_2)u(t-\tau_3)d\tau_1 d\tau_2 d\tau_3}_{y_3(t)} \tag{2}$$

where $y_1(t)$ is the linear and $y_3(t)$ is the cubic polynomial contribution, $u(t)$ is the excitation signal, $h_1(\tau_1)$ and $h_3(\tau_1,\tau_2,\tau_3)$ are the Volterra kernels. These kernels can be written in the frequency domain as:

$$\mathscr{H}_1(\omega_1) = \int_{-\infty}^{\infty} h_1(\tau_1)e^{-j\omega_1\tau_1}d\tau_1 \tag{3}$$

$$\mathscr{H}_3(\omega_1,\omega_2,\omega_3) = \iiint_{-\infty}^{\infty} h_3(\tau_1,\tau_2,\tau_3)e^{-j\omega_1\tau_1}e^{-j\omega_2\tau_2}e^{-j\omega_3\tau_3}d\tau_1 d\tau_2 d\tau_3 \tag{4}$$

The harmonic probing algorithm requires the analytical terms known as higher-order frequency response functions (HOFRFs). Assuming, $\mathscr{H}(\pm j\omega) = \mathscr{H}(\pm\omega)$ and $u(t) = Acos(\omega t) = \frac{A}{2}(e^{j\omega t} + e^{-j\omega t})$, the contributions of the output $y(t)$ can be given by [3]:

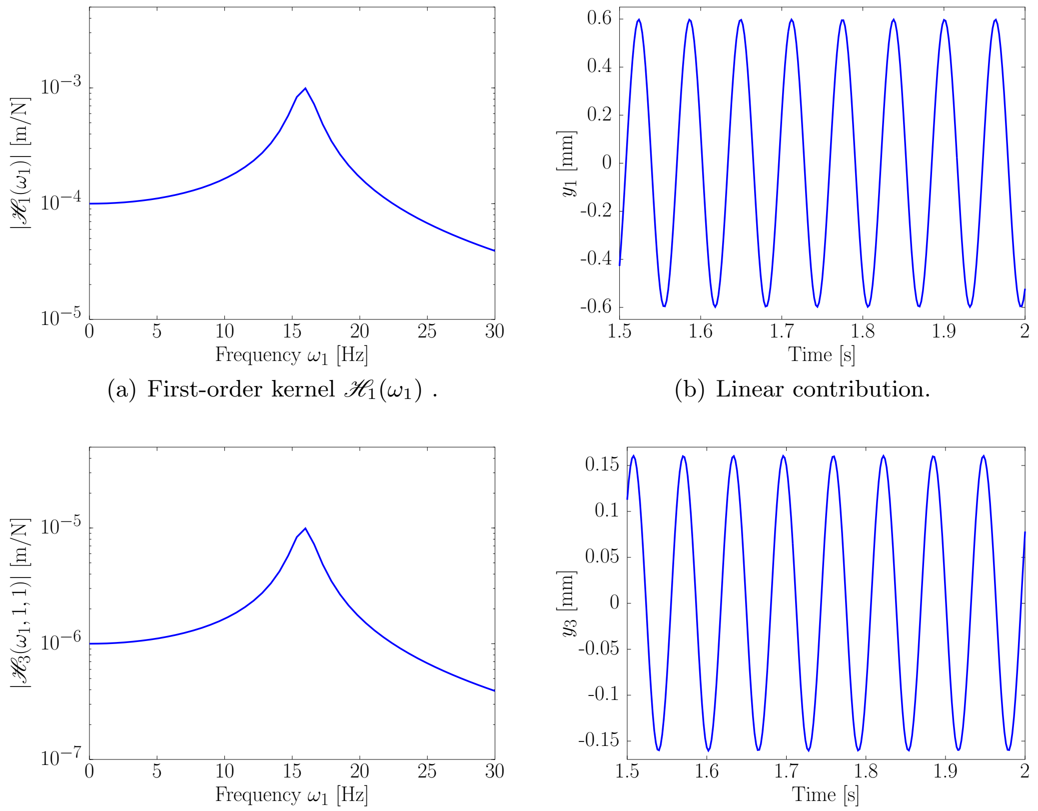
$$y_1(t) = \frac{A}{2}\mathscr{H}_1(\omega)e^{j\omega t} + \frac{A}{2}\mathscr{H}_1(-\omega)e^{-j\omega t} \tag{5}$$

$$\begin{aligned} y_3(t) &= \frac{A^3}{8}\mathscr{H}_3(\omega,\omega,\omega)e^{3j\omega t} + \frac{3A^3}{8}\mathscr{H}_3(\omega,\omega,-\omega)e^{j\omega t} \\ &+ \frac{3A^3}{8}\mathscr{H}_3(\omega,-\omega,-\omega)e^{-j\omega t} + \frac{A^3}{8}\mathscr{H}_3(-\omega,-\omega,-\omega)e^{-3j\omega t} \end{aligned} \tag{6}$$

where the HOFRFs are computed as:

$$\mathscr{H}_1(\omega) = \frac{1}{k_1 + jc\omega - m\omega^2}\ ,\ \mathscr{H}_3(\omega,\omega,\omega) = -k_3\mathscr{H}_1^3(\omega)\mathscr{H}_1(3\omega) \tag{7}$$

It is worth to note that the HOFRFs are directly related with the physical parameters from equation of motion in Eq. (1). In order to illustrate the method, it was employed the parameters of mass $m = 1$ kg, damping $c = 10$ N.s/m, linear stiffness $k_1 = 10^4$ N/m and the cubic stiffness $k_3 = 10^9$ N/m^3. The reference data was obtained by solving numerically Eq. (1) through the Newmark method and the Newton-Raphson algorithm. The sampling rate and number of samples used were $F_s = 500$ Hz and $K = 1024$, respectively, and time duration of 2.0460 seconds. The force applied was $u(t) = Acos(\omega t)$ with amplitude of $A = 0.5981$ N and the excitation frequency in $\omega = 15.9155$ Hz. Figure 1 shows the kernels $H_1(\omega_1)$ and $H_3(\omega_1, 1, 1)$ as well as their contributions y_1 and y_3.



(a) First-order kernel $\mathscr{H}_1(\omega_1)$.

(b) Linear contribution.

(c) Third-order kernel $\mathscr{H}_3(\omega_1, 1, 1)$.

(d) Cubic contribution.

Figure 1: Analysis of the frequency response functions and the respective contributions predicted by harmonic probing method when is applied the input $u(t) = Acos(\omega t)$ with excitation frequency in $\omega = 15.9155$ Hz and amplitude $A = 0.5981$ N.

3 Identification using discrete Volterra-Kautz models

The input and output signals can be obtained through experimental measurements. So, the extraction of the kernels can be performed directly in the discrete-time domain. The idea is to describe the output $y(k)$ of a nonlinear system as:

$$y(k) = \sum_{\eta=1}^{\infty} \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \dots \sum_{n_\eta=0}^{N_\eta} \mathcal{H}_\eta(n_1, n_2 \dots, n_\eta) \prod_{i=1}^{\eta} u(k - n_i) = y_1(k) + y_2(k) + \dots \quad (8)$$

where $\mathcal{H}_\eta(n_1, n_2 \dots, n_\eta)$ are the discrete-time Volterra kernels. Unfortunately, the large number of parameters $N_1, N_2, \dots, N_\eta$ implies in excessive computational requirements and overparametrization effects to identify these kernels. In order to overcome the drawbacks an orthonormal expansion can be used and the kernels mighty be approximated as follows:

$$\mathcal{H}_\eta(n_1, n_2, \ldots, n_\eta) \approx \sum_{i_1=1}^{J_1}\sum_{i_2=1}^{J_2}\cdots\sum_{i_\eta=1}^{J_\eta} \mathscr{B}_\eta(i_1, i_2, \ldots, i_\eta)\prod_{j=1}^{\eta}\psi_{i_j}(n_j;\theta_j) \tag{9}$$

where $\mathscr{B}_\eta(i_1, i_2, \ldots, i_\eta)$ are the coefficients of the orthonormal basis and $\psi_{i_\eta}(n_\eta;\theta_\eta)$ are the Kautz functions that depends of the parameters $\theta_\eta = [z_\eta \ \bar{z}_\eta]$ in which $z_\eta = e^{s_\eta \cdot \Delta t}$ with $\| z_\eta \|$, $\| \bar{z}_\eta \|$<1 (stability condition) where Δt denotes the sampling rate and $s_\eta = -\xi_\eta\varpi_\eta + j\varpi_\eta\sqrt{1-\xi_\eta^2}$ are the parameters in the s-domain. More information about Kautz filters structure and their applicability can be found in [4, 7, 8] . The Kautz filters parameters are obtained minimizing the normalized mean square error function (NMSE):

$$\min F = \frac{\|y(k) - \hat{y}(k)\|}{\|y(k)\|} \tag{10}$$

subject to $\varpi_{(low)\eta} \leq \varpi_\eta \leq \varpi_{(up)\eta}$, $\xi_{(low)\eta} \leq \xi_\eta \leq \xi_{(up)\eta}$ and $\varpi_\eta, \xi_\eta \in \mathbb{R}_+$ where the indexes *up* and *low* represents the upper and lower bounds, respectively. Furthermore, $y(k)$ is the experimental output and $\hat{y}(k)$ is the output estimated by the nonlinear model given by:

$$\hat{y}(k) = \sum_{\eta=1}^{+\infty} y_\eta(k;\theta_\eta) \tag{11}$$

$$\Leftrightarrow \hat{y}(k) = \sum_{\eta=1}^{+\infty}\sum_{i_1=1}^{J_1}\sum_{i_2=1}^{J_2}\cdots\sum_{i_\eta=1}^{J_\eta} \mathscr{B}_\eta(i_1, i_2, \ldots, i_\eta)\prod_{j=1}^{\eta} l_{i_j}(k;\theta_j) \tag{12}$$

This problem can be solved through the least squares approach:

$$\mathbf{\Phi} = (\mathbf{\Lambda}^T\mathbf{\Lambda})^{-1}\mathbf{\Lambda}^T\mathbf{y} \tag{13}$$

where

$$\mathbf{\Lambda} = \left[\begin{array}{ccccccccc} l_1(k) & \ldots & l_{J_1}(k) & l_1^2(k) & l_1(k)l_2(k) & \ldots & l_{J_2}^2(k) & \ldots & l_{J_3}^3(k) & \ldots \end{array}\right],$$

with $l_{i_j}(k;\theta_j) = \sum_{n_j=0}^{V-1}\psi_{i_j}(n_j;\theta_j)u(k-n_i)$ and the vector of unknown terms is given by:

$$\mathbf{\Phi} = \left[\begin{array}{ccccccccc} \mathscr{B}_1(1) & \ldots & \mathscr{B}_1(J_1) & \mathscr{B}_2(1,1) & \ldots & \mathscr{B}_2(J_1, J_2) & \ldots & \mathscr{B}_3(J_1, J_2, J_3) & \ldots \end{array}\right]^T,$$

and $\mathbf{y} = \left[\begin{array}{cccc} y(1) & y(2) & \ldots & y(K) \end{array}\right]^T$ is the output measured with K samples recorded.

In order to illustrate the approach, the benchmark used in section 2 is also used here to compare the estimatives. Firstly, the sequential quadratic programming (SQP) algorithm was employed to solve the optimization problem in Eq. (10) in order to obtain the Kautz filters parameters. To attend this purpose, it were used the lower and upper values $\varpi_{(low)\eta} = 10 \times 2\pi$, $\varpi_{(up)\eta} = 30 \times 2\pi$, $\xi_{(low)\eta} = 0.0010$ and $\xi_{(sup)\eta} = 0.0500$ with $\eta = 1, 3$. The optimal parameters of the Kautz filters were found after 41 iterations and are

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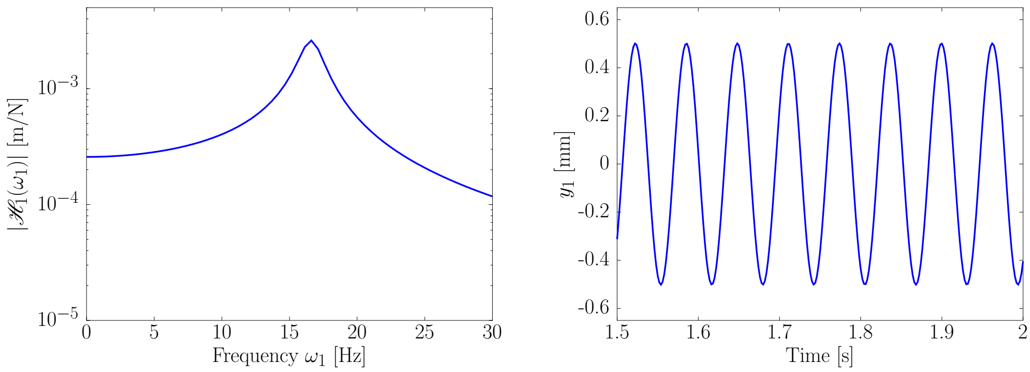
given by $z_1 = 0.9669 + j0.2095$ for the first kernel with $J_1 = 2$ and $z_3 = 0.9511 + j0.2667$ for the third kernel with $J_3 = 2$. The contribution of the second-order kernel was not considered here because the system has only cubic stiffness. In addition, it is important to emphasize that the number of parameters to be estimated was drastically reduced by using the orthonormal expansion once $K \gg J_\eta$. Figure 2 shows the frequency response functions of the Volterra kernels denoted by $\mathscr{H}_1(\omega_1)$ and $\mathscr{H}_3(\omega_3, 1, 1)$ and the respective linear and cubic contributions predicted.



(a) Linear kernel $\mathscr{H}_1(\omega_1)$.

(b) Linear contribution.

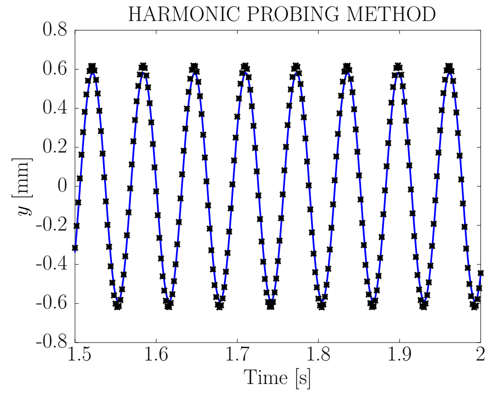
(c) Cubic kernel $\mathscr{H}_3(\omega_1, 1, 1)$.

(d) Cubic contribution.

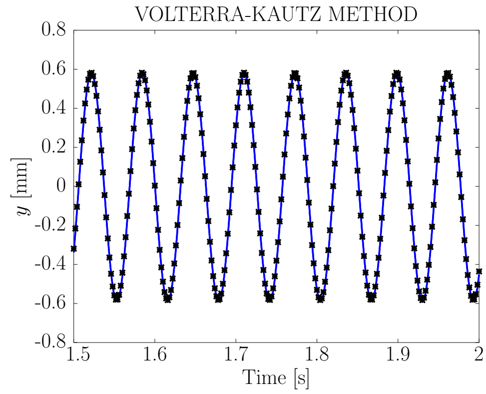
Figure 2: Amplitude of the frequency response functions and the contributions predicted by Volterra-Kautz method when is applied $u(t) = Acos(\omega t)$ with excitation frequency in $\omega = 15.9155$ Hz and amplitude $A = 0.5981$ N.

4 Discussions and comparison between the approaches

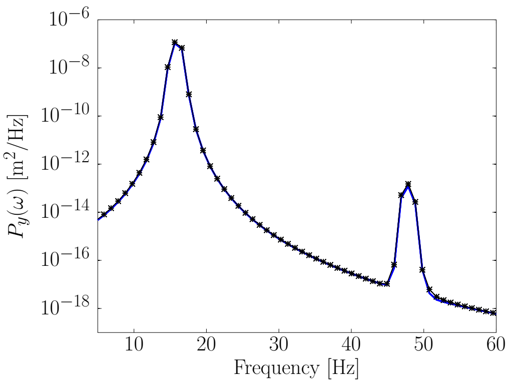
Through the results found its possible to note that the harmonic probing estimatives in Figure 1 present similar behavior compared to the obtained through Volterra-Kautz models as illustrated in Figure 2, but they are slightly different in amplitude. Additionally, the Figures 3(a) and 3(b) show the total output $\hat{y} = y_1 + y_3$ predicted by each method in comparison to the output simulated where it is possible to note a good agreement. Figures 3(c) and 3(d) illustrate the power spectral density (PSD) of the outputs that were calculated using Welch method with Hanning window with 512 samples and 50 % of overlap. In the PSD plot, it is possible to observe that the models were able to detect the presence of fundamental harmonic close to 16 Hz and the respective third-harmonic $3 \times 16 = 48$ Hz.



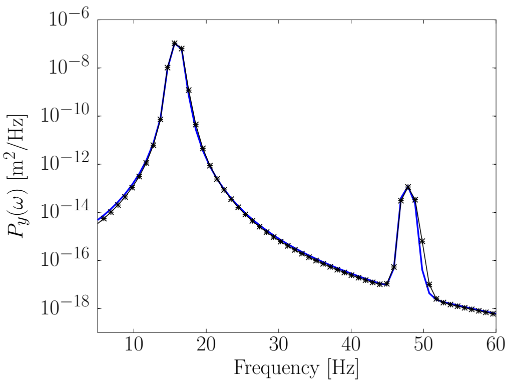
(a) Output simulated (—) × predicted (∗).



(b) Output simulated (—) × predicted (∗).



(c) PSD of the output simulated (—) and predicted by harmonic probing (∗).



(d) PSD of the output simulated (—) and predicted Volterra-Kautz model (∗).

Figure 3: Comparison between the results obtained using the harmonic probing model (left side) and the Volterra-Kautz model (right side).

5 Final remarks and perspectives

The results obtained have shown the applicability of two methods to characterize and detect nonlinear behavior. The harmonic probing method is quite illustrative when are known analytically the differential equations that describe the problem. Now, if there is no knowledge mechanical system and are available only the time series of input and output measures, the Volterra-Kautz models are very useful. Further applications are concerned in modeling and identification of systems with multiple input and multiple output data.

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