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A stochastic analysis of the trajectories of a dry-friction oscillator in a belt: how much sticked and slipping?

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Abstract. This paper investigates the dynamics a simple dry-friction oscillator which is composed of a block, modeled as a particle, connected to a fixed support by a spring. The block moves over a continuous belt that is driven by rollers. The frictional force between the block and the belt is modeled as a Coulomb friction. Due to this friction model, the resulting motion of the block can be characterized into two qualitatively different modes, the stick- and slip-modes, with a non-smooth transition between them. The focus of the paper is to quantify the percent of time in which the block stays in the stick-mode for a non-constant belt velocity and for different values of the friction coefficient from a deterministic and from a stochastic view point.

keywords. Friction-induced vibration, stick-slip, non-smooth system, nonlinear-dynamics, stick duration, stochastic quantification.

1 Introduction

The stick-slip vibrations are self-sustained oscillations induced by dry friction and since the friction can be characterized into two qualitatively different parts (kinetic and static frictions) with a non-smooth transition, the resulting motion also has a non-smooth behavior [2]. Two qualitatively different modes, the stick- and slip-modes, characterize the response of a dry-friction oscillator. We call stick when the relative velocity between the bodies in contact is null in a time-interval and we call slip if the relative velocity is non-zero, or zero in isolated points. As these two modes have a non-smooth transition between them, stick-slip systems belong to the class of non-smooth systems, such as systems with stops, impacts, and hysteresis. The friction non-smooth behavior associated with the absence of a universally accepted friction model give to the friction coefficient a random behavior [1]. The variability of properties of contacting surfaces due to the influence of ambient conditions (such as temperature, humidity, lubrication, state of the surfaces) makes the stochastic approach the ideal way to deal with dry-friction oscillators.

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The main motivation of this paper is to better understand from a deterministic and from a stochastic viewpoint what are the parameters that control the duration of the stick-mode in a very simple dry-friction oscillator. We believe that the theoretical knowledge on the role of these parameters can help in the development of control techniques to reduce the stick duration. These techniques could be implemented in many mechanical systems in which undesirable stick appears. One example of application is on drilling, in which the stick reduces the penetration rate and, following [6], appears during 40% of drilling time.

The focus of the paper is to quantify the percent of time in which the block stays in the stick-mode for a non-constant belt velocity. Numerical experiments were carried on considering deterministic and random belt velocity. The duration of the stick-mode was computed as function of the friction coefficient, μ, and as function of the ratio, r, between the frequency of the belt velocity and the natural frequency of the system.

This paper is organized as follows. Section 2 describes a most simple stick-slip oscillator. It presents also the analytical solutions of the system response. In Section 3, a non constant model of belt velocity is considered and the results of the stick duration as function of μ and r are presented. The construction of the probabilistic model of the uncertain belt velocity is given in Section 4. Statistics of the duration of the stick-mode, are presented in Section 5.

2 Dynamics of the stick-slip oscillator

As explained in the introduction, the system analyzed in this paper is composed by a block connected to a fixed support by a spring. The block moves over a continuous belt that is driven by rollers, as shown in Fig. 1(a). It is assumed that there is friction between the block and the belt which is modeled as a Coulomb friction. Due to this friction model, the resulting motion of the block can be characterized in two qualitatively different modes with a non-smooth transition between them. The first one is the stick-mode, in which the block and belt have the same velocity during an open time interval. The second one is the slip-mode, in which they have different velocities. The position of the block over the belt is represented by x and its equation of motion is

$$m\,\ddot{x}(t) + k\,x(t) = f(t), \tag{1}$$

where m is the mass of the block, k is the spring stiffness and f is the frictional force between the block and the belt. The belt velocity is represented by v and it is assumed that f during the slip-mode depends on the slip velocity, $v - \dot{x}$, as

$$f(t) = n\mu\,\mathrm{sgn}(v(t) - \dot{x}(t)), \tag{2}$$

where n is the normal force exercised by the belt on the block and μ is the friction coefficient. Since we use the simplest friction model, the friction coefficient is assumed to be constant. Thus, during the slip-mode, the value of the frictional force, f, is known. Its absolute value is constant and equal to the maximum friction force, $f_{\max} = \mu n$. During the stick-mode, the block velocity is equal to the belt velocity, i.e. $\dot{x} = v$, and the block acceleration is equal to the belt acceleration, i.e. $\ddot{x} = \dot{v}$. Thus, the equation of motion

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Eq. (1) can be rewritten as

$$m\dot{v} + k\,x(t) = f(t). \tag{3}$$

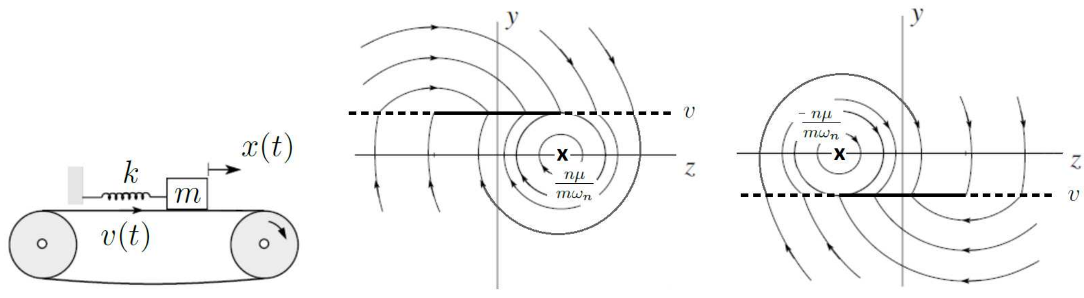
The value of the frictional force during the stick-mode varies and it is confined to the interval $-f_{\max} \leq f \leq f_{\max}$. To better understand the dynamics of a stick-slip oscillator, we start analyzing the case in which the belt velocity is constant in time. This simple configuration has been vastly studied and has known analytical solutions.

2.1 Belt with constant velocity

Considering that the belt velocity and μ are constant in time, represented by v and μ, and introducing the new variables $y = \dot{x}$ and $z = x\,\omega_n$ (where $\omega_n = \sqrt{k/m}$ is the natural frequency of the system), the solutions for the phase paths of the system during the slip-mode are written as

$$\text{when} \quad y > v \qquad y^2 + \left(z + \frac{n\mu}{m\omega_n}\right)^2 = c, \qquad \text{when} \quad y < v \qquad y^2 + \left(z - \frac{n\mu}{m\omega_n}\right)^2 = c. \tag{4}$$

where c is a constant. Thus, given a positive v, the phase diagram of the system, has a single equilibrium point at $(n\mu/m\omega_n, 0)$. Given a negative v, its single equilibrium point is at $(-n\mu/m\omega_n, 0)$, as shown in Figs. 1(b) and 1(c) . In both cases, it is a centre [3, 4]. In Figs. 1(b) and 1(c), the horizontal segment $y = v$ and $|x| \leq \mu n/k$ correspond to the



(a) (b) (c)

Figure 1: (a) Stick-slip oscillator. Phase diagram for the stick-slip oscillator with (b) positive and (c) negative belt velocity.

stick mode, i.e., when the block moves with the belt velocity and the friction force is able to compensate the elastic force to maintain the block put. This mode can happen only before the system reaches the steady-state, because after it, the phase paths are written as a circle with center in the equilibrium point, Eq. 4. Thus, if we draw the phase diagram of a stick-slip oscillator with a constant belt velocity and Coulomb friction, considering only the steady-state solution of the system, there is no stick-mode. More details can be found in [5].

3 Duration of the stick-mode with a deterministic belt velocity

To understand the behavior of the stick-slip oscillator when the belt does not have a constant velocity, we start analyzing a deterministic models. The objectives are to observe the influence of a non-constant belt velocity in the system response in the steady-state, to find the stick- and slip-mode parts of the trajectory and to compute the duration of the stick-mode. For computation, Eq. (1) was integrated in a range of [0.0, 200.0] seconds. For the integration, it was used the function *ode45* of the *Matlab* software, which applies the Runge-Kutta 4th/5th-order method as time-integration scheme with a varying time-step algorithm. The maximal step size is equal to 10^{-4} seconds, and the relative and absolute tolerance are equal to 10^{-4}. The values of the parameters used in all simulations were 1.0 Kg for the block mass, 4.0 N/m for the spring stiffness, 1.0 N for the normal force, 1.0 for the constant friction coefficient and $v_0 = 1.0$ m/s for the modulus of the belt velocity. As initial conditions of the system it was taken $x(0) = 0.0$ m and $\dot{x}(0) = 1.0$ m/s. The non-constant belt velocity analyzed is

$$v(t) = -v_0 \mathrm{sgn}(\sin(\omega_b t)) \,, \tag{5}$$

where v_0 is the amplitude and $2\omega_b$ is the frequency of the sign change of the belt velocity. With this imposed discontinuous belt velocity (a sort of bang-bang control), the block position and velocity in the instants of change of the velocity sign behave as initial conditions to the next time-interval (with size $1/2\omega_b$) in which the belt velocity is constant. Thus, each change of the velocity sign can be understood as a reboot of the system. Due the belt velocity discontinuities, the belt acceleration presents infinite values at some instants. So, this model of belt velocity is a limit case and can not be realized by an experiment. Beside this, since the friction force between the block and the belt has a bounded value, $f_{\max} = \mu n$, the block acceleration will never assume infinite values. Thus, if the block is in the stick-mode in the instant just before the belt discontinuity, it will be obligatorily in the slip-mode in the instant just after the discontinuity. This obligatory change from the stick to slip-mode turns impossible the situation of having just the stick-mode in the system response, i.e., the response must also have a slip-mode part. As explained in the introduction, one of the variables of interest in dry-friction oscillators is the duration of the stick-mode. In the simple system analyzed in this paper, this variable, called t_s, is related with the value of ratio r and the friction coefficient μ. This Section presents results of the duration of the stick-mode as function of r and μ. Considering that the belt velocity is discontinuous, as written in (5), numerical simulations have been carried out combining the following values of the parameters: 193 values for r uniformly selected in the interval $[0.1\,, 2.5]$, and 10 values for μ selected in $[0.05\,, 3.4]$. In each simulation, it was computed the percent of time, t_s, in which occurs the stick-mode during one period of the steady-state solution of the system. Figure 2 shows the obtained results. It can be seen that when μ is low, as $\mu = 0.05$, there is no stick for all values of r. With $\mu = 0.2$, the stick appears in the steady-state solution of the system for r in the interval [1.05 , 1.0875]. The maximal value of t_s is 4.5% and it occurs with $r = 1.0625$. With $\mu = 0.6$, the stick

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appears for r in some intervals of r. The maximal value of t_s, called as $t_s^{\max}$, is 20.23% and it occurs with $r = 1.1875$. Comparing the results obtained with μ equal to 1.0, 1.4 and 1.8, it is verified that the bigger μ is, bigger is the sum of the the measures of the intervals in which $t_s > 0$ and bigger is the maximum value of t_s. For μ equal to 2.2, 2.6, 3.0 and 3.4, the stick appears in an interval of r. Beside this, the bigger μ is, bigger is the measures of this interval. Regarding Fig. 2, it is remarkable that as μ grows, the maximum value

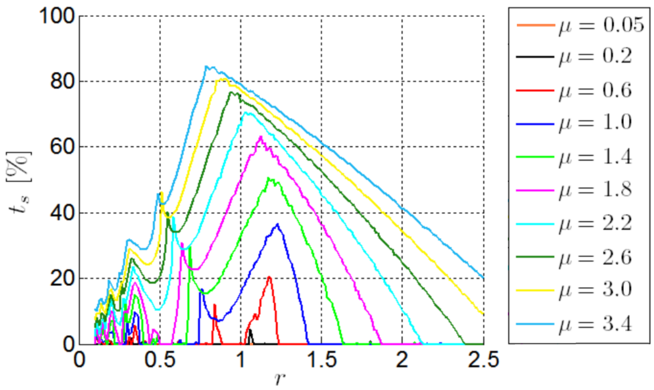

Figure 2: Belt with discontinuous velocity: percent of time in which occurs the stick-mode during one period of the steady-state solution of the system as function of r for different values of μ.

of t_s (called as $t_s^{\max}$) also grows and the value of r in which this maximum value occurs decreases.

4 Stochastic model of the belt velocity

As it was explained in the introduction, to construct a model of the frictional force is not a simple task. To deal with the friction coefficient variability and the friction dependency on the relative velocity of the bodies in contact, we propose to analyze the stick-slip oscillator with a stochastic approach. Considering that the belt velocity is a source of uncertainty in the stick-slip oscillator problem, we propose to treat it as a random process, with parameter t constant by parts, represented by $\mathcal{V}$. We consider that $\mathcal{V}$ assumes only two values, 1.0 m/s and -1.0 m/s. Beside this, we defined an interval $[0\,, T]$ for analysis, the number of changes of the velocity sign of $\mathcal{V}$ in this interval is given by a random variable P with Poisson distribution. Thus, for $p = 0,\ 1,\ 2,\ \ldots$, the probability mass function of P is given by

$$Pr(P = p) = \frac{\lambda^p \, e^{-\lambda}}{p!}\,. \tag{6}$$

where λ is the mean. With this model, each realization of $\mathcal{V}$ is a periodic function. Consequently, the frequency ω_b of the belt velocity becomes a random variable Ω_b, written as $\Omega_b = (P+1)(2T)$. The mean of Ω_b is $\underline{\Omega_b} = (\lambda+1)/(2T)$. The ratio between the frequency of the belt velocity and the natural frequency of the system becomes a random variable R, written as $R = \Omega_b/\omega_n$.

5 Numerical simulations of the stick-slip oscillator with random belt velocity

As it was assumed that the belt velocity is uncertain, modeled as a random process $\mathcal{V}$, the response of the stochastic stick-slip oscillator is a random process with parameter t [7]. Thus, the equation of motion of the system, Eq. 1, becomes a stochastic differential equation and the percentage of time in which the stick-mode occurs becomes a random variable T_s. Statistics of T_s were estimated by the Monte Carlo simulation method using 200 independent realizations of the random variable P following its probability distribution. To observe the influence of the friction coefficient and of the mean of R in the statistics of T_s, Monte Carlo simulations have been carried out combining the following values of the parameters: 49 values for $\underline{R}$ uniformly selected in the interval $[0.1\,,2.5]$, and 8 values for μ uniformly selected in the interval $[0.6\,,3.4]$). Figure 3 shows the mean of T_s as function of $\underline{R}$ for different values of μ. Comparing it with Fig. 2, it is verified that $\underline{T_s}$ behaves as t_s for the case of belt with harmonic velocity.

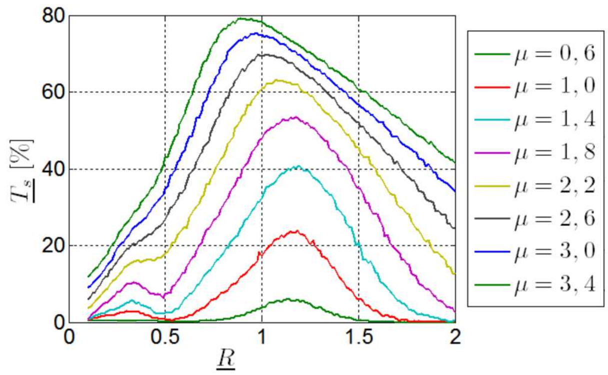

Figure 3: Random belt velocity: $\underline{T_s}$ as function of $\underline{R}$ for different values of μ.

6 Conclusions

In the deterministic analysis of the stick-slip oscillator with a constant belt velocity and the same surface conditions along its length, it is verified that considering only the steady-state solution, there is no stick-mode, when the Coulomb's model is used. The block oscillates around an equilibrium point related with the normal force exerted on the block, the friction coefficient and the natural frequency of system.

When the belt has a different imposed periodic velocity, as a discontinuous velocity, it was verified, by numerical simulations, that the stick-mode can occur in the steady-state of the system, depending on the friction coefficient and on the ratio r between the

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frequency of the belt velocity and the natural frequency of the system. Beside this, it was also verified that the duration of the stick-mode varies a lot with the system configuration. Depending on the model of belt velocity, on μ and on r, the stick-mode can take any value, from a large fraction of the duration of the solution, reaching values up to 100%, or may even be null.

In the stochastic analysis, the belt velocity was modeled it as a random process constant by parts, for which defined an interval $[0\,,T]$, the number of changes of the velocity sign of the belt velocity in this interval is given by a random variable P with Poisson distribution. By Monte Carlo simulations, the mean of the duration of the stick-mode was computed for different values of ratio $\underline{R}$ and μ. Comparing these statistics with the results of deterministic simulations for the case of belt with discontinuous velocity, it is verified that in relation to the mean of the stick duration, deterministic and stochastic systems have similar behavior.

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