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Equilibrium Conditions for Tethered Nanosatellite Constellations

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This expanded summary describes a tethered spacecraft system modeled in three-dimensional problem with six smaller bodies connected by a cable of negligible mass and rigid. This allows the model to be described by a Keplerian motion. Our goal is analyzing equilibrium solutions of the tether using a control law for the cable size. The solutions are found assuming a control law in function of independent variables eccentricity (e) and true anomaly (ν) that describe the orbital dynamic of the problem. For uniform rotation of the variation of the mass center of system and the constant angle rotation planar were found continuous periodic solutions for different values of e and ν.

The problem involving several bodies connected by cables has been studied by several authors [1–3], there was a demand to connect structures such as subsatellites, optimize costs in space missions, transport and even space debris collection. In this context, six bodies were connected by a cable, which moves their center of mass in the plane and around the main body, with the objective of analyzing their dynamics and their orbital behavior, under the influence of the space environment, for future applications in orbital maneuvers of close approach in planets and asteroids.

Mathematical model

The model is a dumbbell-like system, with the six points mass (m_i) connected with a rigid and massless tether. The system's center of mass describes a Keplerian orbit fixed on an inert axis in primary body's center [3].

The angle ν is the tether true anomaly around primary body. The variable ρ is the distance from primary body to the center of mass (CM), l_i are tether lengths from the points mass m_i to CM, respectively. φ is the angle between the tether plane projection and ρ, and it describes the plane of motion. ψ is the angle between the tether and primary body plane. Since the model chosen is based on points masses and a massless tether, this allows the model to be described by Keplerian motion, where p is the focal parameter, e and ν:

$$\left\{\begin{matrix} l_1 = \frac{m_1 lx}{m_1+m_2} & l_2 = \frac{m_2 lx}{m_1+m_2} & l_1 + l_2 = lx \\ l_3 = \frac{m_3 ly}{m_3+m_4} & l_4 = \frac{m_4 ly}{m_3+m_4} & l_3 + l_4 = ly \\ l_5 = \frac{m_5 lz}{m_5+m_6} & l_6 = \frac{m_6 lz}{m_5+m_6} & l_5 + l_6 = lz \end{matrix}\right\} \tag{1}$$

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The true anomaly, the masses of the bodies (m_i) are the same. For the coordinates system the components of center of mass position vector (x_0, y_0, z_0) are:

$$\left\{\begin{array}{ccc} x_0 = \rho\cos(\nu) & y_0 = \rho\sin(\nu) & z_0 = 0 \\ x_1 = x_0 + l_1\cos(\nu+\varphi)\cos(\psi) & y_1 = y_0 + l_1\sin(\nu+\varphi)\cos(\psi) & z_1 = l_1\sin(\psi) \\ x_2 = x_0 - l_2\cos(\nu+\varphi)\cos(\psi) & y_2 = y_0 - l_2\sin(\nu+\varphi)\cos(\psi) & z_2 = -l_2\sin(\psi) \\ x_3 = x_0 - l_3\sin(\nu+\varphi)\cos(\psi) & y_3 = y_0 + l_3\cos(\nu+\varphi)\cos(\psi) & z_3 = l_3\sin(\psi) \\ x_4 = x_0 + l_4\sin(\nu+\varphi)\cos(\psi) & y_4 = y_0 - l_4\cos(\nu+\varphi)\cos(\psi) & z_4 = -l_4\sin(\psi) \\ x_5 = x_0 - l_5\cos(\nu+\varphi)\sin(\psi) & y_5 = y_0 - l_5\sin(\nu+\varphi)\sin(\psi) & z_5 = l_5\cos(\psi) \\ x_6 = x_0 + l_6\cos(\nu+\varphi)\sin(\psi) & y_6 = y_0 + l_6\sin(\nu+\varphi)\sin(\psi) & z_6 = -l_6\cos(\psi) \end{array}\right\} \tag{2}$$

The Lagrange Equations of motion is ordinary differential equations, which describe the motions mechanical systems under the action of forces, can be obtained by $L = T - V$. Where T is the kinetic energy and V is the potential energy. For the following analysis, the generalized coordinates are φ and ψ. The system is only under the gravity-gradient. These forces make a conservative system. Two equations can be obtained based on those coordinates.

Defining φ or $l(lx, ly, lz)$ its is possible to control the system, as can be seen in [2, 4, 5]. φ was defined and then and consecutively obtained the l behavior. The motion in the spherical coordinates, ψ, φ, ν and l, where lx, ly, lz represents the cable length, in the corresponding direction x, y and z, $()' = \frac{d}{d\nu}$, ν and φ in-plane and ψ out-plane rotation.

$$\begin{aligned} &4(1+e\cos(\nu))(\varphi'+1)\cos^2(\psi)(lxlx'+lyly') + 4lzlz'(\varphi'+1)\sin^2(\psi)(1+e\cos(\nu)) \\ &+ lz(\nu)^2\left(\sin^2(\psi)(2\varphi''(1+e\cos(\nu)) - 4e\sin(\nu) + 3\sin(2\varphi))\right. \\ &\left.+ 2\varphi'\left(\psi'\sin(2\psi)(1+e\cos(\nu)) - 2e\sin(\nu)\sin^2(\psi)\right) + 2\psi'\sin(2\psi)(1+e\cos(\nu))\right) = \cos(\psi)\left(2\left(lx^2\right.\right. \\ &\left.+ ly^2\right)\left(2(\varphi'+1)(\psi'\sin(\psi)(1+e\cos(\nu)) + e\sin(\nu)\cos(\psi)) - \varphi''\cos(\psi)(1+e\cos(\nu))\right) \\ &\left.+ 3\left(ly^2 - lx^2\right)\sin(2\varphi)\cos(\psi)\right) \end{aligned} \tag{3}$$

$$\begin{aligned} &2\left(lx^2+ly^2+lz^2\right)((1+e\cos(\nu))\psi'' - 2e\sin(\nu)\psi') \\ &+ (\varphi'+1)^2\sin(2\psi)(1+e\cos(\nu))\left(lx^2+ly^2-lz^2\right) + 4\psi'(1+e\cos(\nu))(lxlx'+lyly'+lzlz') \\ &+ 3\sin(2\psi)\left(lx^2\cos^2(\varphi) + ly^2\sin^2(\varphi)\right) - 3lz^2\cos^2(\varphi)\sin(2\psi) = 0 \end{aligned} \tag{4}$$

The uniform rotations of a dumbbell and with several possibilities are considered in the present study, as well as the stability analysis and the viable control laws.

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