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Numerical critical time determination of the Kelvin Helmholtz instability

Gaspar Gonzalez[1]
National University of Asuncion, FACEN San Lorenzo, PY
Inocencio Ortiz[2]
National University of Asuncion, FP San Lorenzo, PY
Christian E. Schaerer[3]
National University of Asuncion, LCCA San Lorenzo, PY

1 Introduction

Kelvin-Helmholtz instability (KHI) is a type of hydrodynamic instability that appears at the interface of two immiscible fluids due to different (tangential) velocities of the streams. In this work we consider two immiscible ideal fluids and implement a simulation to determine the time at which the instability begins to appear.

To validate our simulation, we contrast it with both theoretical analysis and experimental results found in the literature. For the theoretical part, the results area contrasted with results based on normal modes [1], while for experimental results, it is considered results obtained in [2].

2 Methodology and Results

The general condition for the KHI to develop between two immiscible and incompressible fluids, neglecting surface tension is given by

$$\frac{(\rho_2 - \rho_1)}{(\rho_2 + \rho_1)} \frac{g}{k} < \frac{\rho_2 \rho_1}{(\rho_2 + \rho_1)^2} (U_2 - U_1)^2 , \tag{1}$$

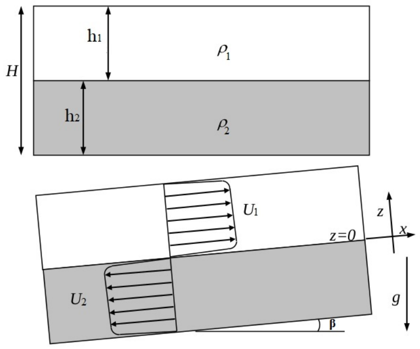
where ρ is the density, g is gravity acceleration, k is the wave number, U is the velocity of the fluid [1]. According to [2], in the experimental setting, as depicted in Figure 1(a), the KHI starts when

$$t > t_c = \sqrt{\frac{\lambda (\rho_1 + \rho_2)^3}{8\pi g \sin^2(\beta) (\rho_2 - \rho_1) \rho_1 \rho_2}}, \tag{2}$$

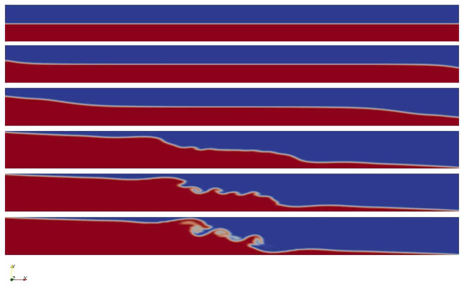
where λ is the wavelength of the disturbance and β is the angle of inclination of the box. We denote by t_c the *critical time*. The *critical time* is measured prev the characteristic peaks of the KHI appear, well that means that the instability will occur for a time t greater than t_c measured. The simulation was implemented in OpenFoam, using the interFoam solver. It is a two-dimensional simulation, the walls of the box have no movement and the fluids are initially static, clarifying that

[1]Email: gaspar.ag.acosta@gmail.com
[2]Email: inortiz08@gmail.com; corresponding author.
[3]Email: chris.schaerer@gmail.com; team leader.

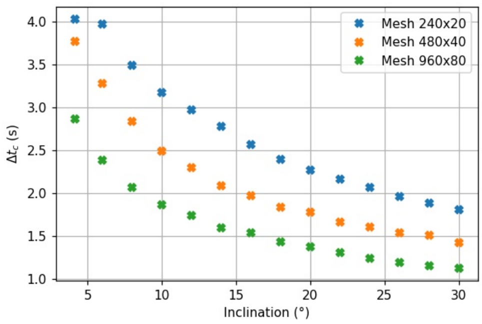
despite the fact that the domain appears in a horizontal position in the simulation the effect of gravity is taken into account. Figure 1(b) shows the temporal evolution of the simulation for the meshing of 240×20, in this case with $\beta = 6^o$, and time in seconds corresponding to $t = 0$, $t = 3$, $t = 5$, $t = 6$, $t = 6.5$ (characteristic peaks of instability), and $t = 7$, respectively. In Figure 1(c), t_{c_s} is the critical simulation time and t_{c_t} (determined by equation (2)) is the theoretical critical time.



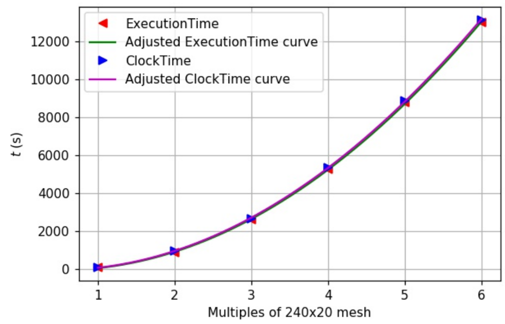
(a) Experiment mounting scheme, [2]



(b) Time evolution of the simulation



(c) Critical time variation $\Delta t_c = t_{c_s} - t_{c_t}$



(d) Computation time

Figure 1: Mounting the experiment and results obtained

3 Conclusions

The critical time variation $\Delta t_c = t_{c_s} - t_{c_t}$ decreases as the mesh refinement increases (see Figure 1(c)). This suggest that we can get better agreement by refining the mesh. On the other hand, as seen in Figure 1(d), the computation time until capturing the phenomenon of instability increases quadratically with the mesh refinement.

References

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