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Correlation Analysis of Intuitionistic Fuzzy Connectives

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Abstract. This paper studies the correlation between Atanassov's intuitionistic fuzzy sets (A-IFSs) obtained as image of strong intuitionistic fuzzy negations. We consider the action of strong fuzzy negations in order to verify the conditions under which the correlation coefficient related to such A-IFSs and their corresponding conjugate constructions are obtained.

Keywords. Correlation coefficient, Intuitionistic Fuzzy Logic, Fuzzy Negations, Strong Fuzzy Negations

1 Introduction

The Atanassov's intuitionistic fuzzy logic (A-IFLs) comprises a generalization of multi-valued fuzzy logic by taking into account the membership and non-membership functions which are not necessarily complementary with respect to Atanassov's intuitionistic fuzzy sets (A-IFSs). This approach leads to a great numbers of studies. See, e.g., the index of intuitionist of an element remaining the hesitation between the membership and non-membership degree, relating similarity measure to analyse the consensus of an expert preference in group decision making; dealing with similarity measure to indicate the similar degree of two intuitionistic fuzzy sets; and analysing the entropy of A-IFSs and describing its fuzziness degree. All of them are closely connected with the correlation coefficient between two intuitionistic fuzzy sets.

In our previous work [1,2], it is shown that the conjugate of an interval-valued Atanassov's intuitionistic fuzzy negation not only preserves the main properties of its corresponding fuzzy negation but also of two other ones, the intuitionistic fuzzy negation and interval-valued fuzzy negation. Moreover, we introduce an extension of the intuitionistic fuzzy index as well as study basic concepts related to the correlation index in the sense of [3],

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which are discussed in terms of fuzzy connectives, by considering the Atanassov's Intuitionistic Fuzzy Logic.

Thus, following the approach introduced in [3], this article considers the problem of defining the correlation coefficient between two A-IFSs, as conceived in [4] and [5]. Additionally, new properties of correlation coefficients can be obtained from intuitionistic fuzzy negations, which can be applied to the fuzzy data analysis and classification in prediction, diagnosis and decision making. We show that by interpreting an A-IFS as the image of a fuzzy connective on U, we are able to derive a simple correlation coefficient between two related A-IFSs. Such operators are also obtained by the action of strong fuzzy intuitionistic negations. The correlation coefficients of such classes of A-IFSs, their corresponding conjugate and dual constructions are also analysed.

This paper is organized as follows: Section 3 brings the main concepts of correlation coefficient from intuitionistic fuzzy logic. In Section 4, the study includes the main results based on correlation coefficient obtained by strong fuzzy negations. Finally, conclusions and further work are discussed in Section 5.

2 Preliminaries

This section considers a non-empty, finite and enumerable universe $\mathcal{U} = \{u_1, \ldots, u_n\}$, such that for an A-IFS A based on χ, the membership and non-membership functions $\mu_A, \nu_A : \mathcal{U} \to U$ are related by the inequality $\mu_A(u_i) + \nu_A(u_i) \leq 1$, for all $i \in \mathbb{N}_n = \{1, 2, \ldots, n\}$. Additionally, the intuitionistic fuzzy index is given as follows:

$$\pi_A(u_i) = 1 - \mu_A(u_i) - \nu_A(u_i) \tag{1}$$

is called the intuitionistic fuzzy index (IFIx) of an A-IFS A. The set of all above related A-IFSs A is denoted by $\mathcal{C}(A_I)$.

Let $\tilde{U} = \{\tilde{x}_i = (x_{i1}, x_{i2}) \in U^2 : x_{i1} + x_{i2} \leq 1\}$ be the set of intuitionistic fuzzy values such that $\tilde{x}_i$ is a pair of membership and non-membership degrees of an element $u_i \in \mathcal{U}$, i.e. $(x_{i1}, x_{i2}) = (\mu_A(u_i), \nu_A(u_i))$, respectively. And, the related IFIx the degree of hesitance of an A-IFS A is given as $\pi_A(u_i) = x_{i3} = 1 - x_{i1} - x_{i2}$.

The projections $l_{\tilde{U}^n}, r_{\tilde{U}^n} : \tilde{U}^n \to U^n$ are given by:

$$\begin{aligned} l_{\tilde{U}^n}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n) &= (x_{11}, x_{21}, \ldots, x_{n1}) && (2)\\ r_{\tilde{U}^n}(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n) &= (x_{12}, x_{22}, \ldots, x_{n2}) && (3) \end{aligned}$$

The order relation $\leq_{\tilde{U}}$ on $\tilde{U}$ is defined as: $\tilde{x} \leq_{\tilde{U}} \tilde{y} \Leftrightarrow x_1 \leq y_1$ and $x_2 \geq y_2$. Additionally, $\tilde{0} = (0, 1) \leq_{\tilde{U}} \tilde{x} \leq_{\tilde{U}} (1, 0) = \tilde{1}$, for all $\tilde{x} \in \tilde{U}$, see details in [6].

3 Correlation from intuitionistic fuzzy logic

The correlation between intuitionistic fuzzy sets obtained by action intuitionistic fuzzy connectives are studied, by considering the following denotation

$$(\mu_A(u_1), \mu_A(u_2), \ldots, \mu_A(u_n)) = (x_{11}, x_{21}, \ldots, x_{n1});$$
$$(\nu_A(u_1), \nu_A(u_2), \ldots, \nu_A(u_n)) = (x_{12}, x_{22}, \ldots, x_{n2});$$
$$(\pi_A(u_1), \pi_A(u_2), \ldots, \pi_A(u_n)) = (x_{13}, x_{23}, \ldots, x_{n3}).$$

and two corresponding classes of the quasi-arithmetic means:

(i) the **arithmetic mean**, performed over the all intuitionistic fuzzy values of an A-IFS A as follow:

$$m(x_{1k}, x_{2k}, \ldots, x_{nk}) = \frac{1}{n}\sum_{i=1}^{n} x_{ik}, \text{ for each } \quad k \in \{1,2,3\};$$

(ii) the **quadratic mean**, performed over the difference between each intuitionistic fuzzy value of an A-IFS A and the corresponding arithmetic mean of all its values, as described in the following:

$$m_2(x_{1k}, x_{2k}, \ldots, x_{nk}) = \sqrt{\sum_{i=1}^{n}\left(x_{ik} - \frac{1}{n}\sum_{j=1}^{n} x_{jk}\right)^2}; \text{ for each } \quad k \in \{1,2,3\}.$$

Thus, the quotient between product values obtained by taking two sums performed over such classes of quasi-arithmetic means extends the correlation definition to the Atanassov-intuitionistic fuzzy approach.

According with [5], the **correlation between A-IFSs** A **and** B in $\mathcal{C}(A)$ is given as:

$$\mathbf{C}(A,B) = \frac{1}{3}(C_1(A,B) + C_2(A,B) + C_2(A,B)) \tag{4}$$

wherever the following holds:

$$C_1(A,B) = \frac{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)\left(y_{i1} - \frac{1}{n}\sum_{j=1}^{n} y_{j1}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i1} - \frac{1}{n}\sum_{j=1}^{n} y_{j1}\right)^2}}$$

$$C_2(A,B) = \frac{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)\left(y_{i2} - \frac{1}{n}\sum_{j=1}^{n} y_{j2}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i2} - \frac{1}{n}\sum_{j=1}^{n} y_{j2}\right)^2}}$$

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$$C_3(A,B) = \frac{\sum_{i=1}^{n}\left(x_{i3} - \frac{1}{n}\sum_{j=1}^{n} x_{j3}\right)\left(y_{i3} - \frac{1}{n}\sum_{j=1}^{n} y_{j3}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i3} - \frac{1}{n}\sum_{j=1}^{n} x_{j3}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i3} - \frac{1}{n}\sum_{j=1}^{n} y_{j3}\right)^2}}$$

Based on [5], the correlation coefficient $\mathbf{C}(A,B)$ in Eq.(4) considers both factors: the amount of information expressed by the membership and non-membership degrees expressed by $C_1(A,B)$ and $C_2(A,B)$, respectively; and the reliability of information expressed by the hesitation margins in $C_3(A,B)$. Additionally, for fuzzy data, these expressions just make sense for A-IFS variables whose values vary and avoid zero in the denominator.

By [5], the correlation coefficient $\mathbf{C}(A,B)$ fulfils the following properties:

(i) $\mathbf{C}(A,B) = \mathbf{C}(A,B)$; **(ii)** If $A = B$ then $\mathbf{C}(A,B) = 1$; **(iii)** $-1 \leq \mathbf{C}(A,B) \leq 1$.

4 Main Results

This section studies the correlation related to A-IFSs obtained by action of a IFC, which means a function $F : \tilde{U}^n \to \tilde{U}$. For all $u_i \in \mathcal{U}$, $i \in \mathbb{N}_n$ consider the A-IFS given as

$$A_F = \{(u_i, \mu_F(u_i), \nu_F(u_i)) : \mu_F(u_i) + \nu_F(u_i) \leq 1\}. \tag{5}$$

4.1 Correlation from intuitionistic fuzzy negations

A_N denotes the intuitionistic fuzzy set obtained as image of an intuitionistic fuzzy negation N related to an A-IFS A, whose definition, for $u_i \in \mathcal{U}, i \in \mathbb{N}_n$ is given by:

$$A_N = \{(\mu_{A_N}(u_i), \nu_{A_N}(u_i)) \in \tilde{U} : \mu_{A_N}(u_i) \leq N_S(\nu_{A_N}(u_i))\}. \tag{6}$$

When N is a N_S-representable SIFN, an A-IFS A_N in Eq.(6) can also be given by the following expression: $A_N = \{\tilde{x} = (x_{i2}, x_{i1}) \in \tilde{U} : x_{i1} + x_{i2} \leq 1\}$, that also means

$$A_N = \{(\nu_A(u_i), \mu_A(u_i)) : \mu_{A_N}(u_i) \leq N_S(\nu_{A_N}(u_i)))\}. \tag{7}$$

Proposition 4.1. *Let N be a SIFN and A_N and B_N be the intuitionistic fuzzy sets expressed according with Eq. (7), whenever A and B are A-IFSs on $\mathcal{U}$. Then, the following holds:*

$$C_1(A,B_N) = C_2(A_N,B); \qquad C_2(A,B_N) = C_1(A_N,B); \qquad C_3(A,B_N) = C_3(A_N,B). \tag{8}$$

Proof.

$$C_1(A,B_N) = \frac{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)\left(y_{i2} - \frac{1}{n}\sum_{j=1}^{n} y_{j2}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i2} - \frac{1}{n}\sum_{j=1}^{n} y_{j2}\right)^2}} = C_2(A_N,B)$$

$$C_2(A, B_N) = \frac{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)\left(y_{i1} - \frac{1}{n}\sum_{j=1}^{n} y_{j1}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i1} - \frac{1}{n}\sum_{j=1}^{n} y_{j1}\right)^2}} = C_1(A_N, B)$$

$$C_3(A, B_N) = \frac{\sum_{i=1}^{n}\left(x_{i3} - \frac{1}{n}\sum_{j=1}^{n} x_{j3}\right)\left(y_{i3} - \frac{1}{n}\sum_{j=1}^{n} y_{j3}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i3} - \frac{1}{n}\sum_{j=1}^{n} x_{j3}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i3} - \frac{1}{n}\sum_{j=1}^{n} y_{j3}\right)^2}} = C_3(A_N, B)$$

Therefore, Proposition 4.1 holds. □

Corollary 4.1. *Let N be a SIFN and A_N and B_N be the intuitionistic fuzzy sets expressed according with Eq. (7), whenever A and B are A-IFSs on $\mathcal{U}$. Then, the following holds:*

$$\mathbf{C}(A, B_N) = \mathbf{C}(A_N, B). \tag{9}$$

Proof. Straightforward from Proposition4.1. □

Proposition 4.2. *Let N be a SIFN and A_N and B_N the intuitionistic fuzzy sets expressed according with Eq. (7), whenever A and B are A-IFSs on $\mathcal{U}$. Then, the following holds:*

$$C_1(A_N, B_N) = C_2(A, B); \quad C_2(A_N, B_N) = C_1(A, B); \quad C_3(A_N, B_N) = C_3(A, B). \tag{10}$$

Proof.

$$C_1(A_N, B_N) = \frac{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)\left(y_{i2} - \frac{1}{n}\sum_{j=1}^{n} y_{j2}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i2} - \frac{1}{n}\sum_{j=1}^{n} y_{j2}\right)^2}} = C_2(A, B)$$

$$C_2(A_N, B_N) = \frac{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)\left(y_{i1} - \frac{1}{n}\sum_{j=1}^{n} y_{j1}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i1} - \frac{1}{n}\sum_{j=1}^{n} y_{j1}\right)^2}} = C_1(A, B)$$

$$C_3(A_N, B_N) = \frac{\sum_{i=1}^{n}\left(x_{i3} - \frac{1}{n}\sum_{j=1}^{n} x_{j3}\right)\left(y_{i3} - \frac{1}{n}\sum_{j=1}^{n} y_{j3}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i3} - \frac{1}{n}\sum_{j=1}^{n} x_{j3}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i3} - \frac{1}{n}\sum_{j=1}^{n} y_{j3}\right)^2}} = C_3(A, B)$$

Therefore, Proposition 4.2 holds. □

Corollary 4.2. *Let N be a SIFN and A_N and B_N be the intuitionistic fuzzy sets expressed according with Eq. (7), whenever A and B are A-IFSs on $\mathcal{U}$. Then, the following holds:*

$$\mathbf{C}(A, B) = \mathbf{C}(A_N, B_N). \tag{11}$$

Proof. Straightforward from Proposition 4.2. $\square$

Proposition 4.3. *Let N be a SIFN and A_N be the intuitionistic fuzzy set expressed in Eq. (7), whenever A is an A-IFS on $\mathcal{U}$. The correlation between A and A_N is given by*

$$\mathbf{C}(A, A_N) = \frac{1}{3}(2\,C_1(A, A_N) + 1) = \frac{1}{3}(2\,C_2(A, A_N) + 1). \tag{12}$$

Proof. By Eq. (4), the following holds:

$$C_1(A, A_N) =$$

$$= \frac{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)\left(y_{i1} - \frac{1}{n}\sum_{j=1}^{n} y_{j1}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i1} - \frac{1}{n}\sum_{j=1}^{n} y_{j1}\right)^2}} = \frac{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)^2}\sqrt{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)^2}}$$

$$= \frac{\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)^2}\sqrt{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)^2}} = \frac{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i1} - \frac{1}{n}\sum_{j=1}^{n} x_{j1}\right)^2}\sqrt{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)^2}}$$

$$= \frac{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)\left(y_{i2} - \frac{1}{n}\sum_{j=1}^{n} y_{j2}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{i2} - \frac{1}{n}\sum_{j=1}^{n} x_{j2}\right)^2}\sqrt{\sum_{i=1}^{n}\left(y_{i2} - \frac{1}{n}\sum_{j=1}^{n} y_{j2}\right)^2}} = C_2(A, A_N).$$

Moreover, by Eq. (7), the equalily $\pi_A(u_i) = \pi_{A_N}(u_i)$ results that $x_{i3} = y_{i3}$, for all $i \in \mathbf{N}_n$, implying the following:

$$C_3(A, A_N) = \frac{\sum_{i=1}^{n}\left(x_{i3} - \frac{1}{n}\sum_{j=1}^{n} x_{j3}\right)^2}{\sum_{i=1}^{n}\left(x_{i3} - \frac{1}{n}\sum_{j=1}^{n} x_{j3}\right)^2} = 1.$$

Therefore, Eq. (12) holds. $\square$

Example 1. *Let $A = \{((0.2, 0.7), (0.3, 0.5)\}$ and $B = \{((0.3, 0.6), (0.1, 0.4)\}$ be A=IFSs. Thus, we have the following pairs: $(x_{11}, x_{21}) = (0.2, 0.3)$; $(x_{12}, x_{22}) = (0.7, 0.5)$; $(x_{13}, x_{23}) = (0.1, 0.2)$; $(y_{11}, y_{21}) = (0.3, 0.1)$; $(y_{12}, y_{22}) = (0.6, 0.4)$; $(y_{13}, y_{23}) = (0.1, 0.5)$. Moreover, their corresponding arithmetic means and quadratic means are given as:*

$$\begin{array}{ll} m(x_{11}, x_{21}) = m(y_{12}, y_{22}) = 0.25 & m_2(x_{11}, x_{21},) = m_2(y_{12}, y_{22}) = \sqrt{0.005} \\ m(x_{12}, x_{22}) = m(y_{11}, y_{21}) = 0.6 & m_2(x_{12}, x_{22},) = m_2(y_{11}, y_{12}) = \sqrt{0.02} \\ m(x_{13}, x_{23}) = m(y_{13}, y_{23}) = 0.15 & m_2(x_{13}, x_{23}) = m_2(y_{13}, y_{23}) = \sqrt{0.005} \end{array}$$

Then, the following holds:

$$C_1(A, A_N) = C_2(A, A_N) = \frac{-(0.1)^2}{\sqrt{0.005}\sqrt{(0.02)}} = \frac{-0.01}{0.01} = -1;$$
$$C_3(A, A_N) = \frac{(-0.005 \cdot -0.005) + (0.005 \cdot 0.005)}{\sqrt{0.005}\sqrt{0.005}} = \frac{0.005}{0.005} = 1.$$

Result correlation coefficient: $\mathbf{C}(A, A_N) = \frac{1}{3}(-1 - 1 + 1) = -\frac{1}{3}$*. Analogously, one can prove that* $\mathbf{C}(B, B_N) = \frac{1}{3}(1+1+1) = 1$*. Moreover, the membership values of the elements in* A *increase whereas the membership values of elements in* B *decrease resulting in:*
$\mathbf{C}(A, B) = \frac{1}{3}(-1 + 1 + 1) = \frac{1}{3}$ $\quad$ $\mathbf{C}(A_N, B_N) = \frac{1}{3}(1 - 1 + 1) = \frac{1}{3}$

5 Current Work

In this paper the formulation of correlation coefficient was introduced, this formulation has been applied to an intuitionistic fuzzy set and its negation. Moreover, new properties of the coefficient correlation are verified by A-IFSs obtained from strong fuzzy negations.

Further work intends to extend these correlation formulations of A-IFSs to other fuzzy connectives and representable automorphisms, also applying these new formulation correlation to the set of input data.

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