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On Calculating the Transient Response of Unrestrained Structures with Fourier Analysis

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1 Introduction

The transient displacement response of a free, unrestricted structure is in general an infinite-energy signal, due to the existence of rigid body (RB) motion, and thus cannot be represented by a Fourier Transform. Therefore most of the spectral approaches such as analytical solutions or the Spectral Element Method cannot be used to obtain the response of such structures, in which case the Finite Element Method (FEM) is widely recurred to. In this paper an alternative strategy is proposed in which one may remove the RB motion from the frequency domain solution and then take its Inverse Fourier Transform to obtain the signal in the time domain. The RB motion contribution may be obtained separately and added to the time domain solution. As an example the method is used to obtain the transient response of a free-free homogeneous rod and the result is then compared to the FEM solution. All the computations were performed in the MATLAB® environment.

2 Methodology and results

Considering a free-free straight homogeneous rod with length L, cross-sectional area A, Young's modulus E, density ρ and viscous damping coefficient η, impacted by a compression pulse $F(t)$ on its left end, the partial differential equation and its boundary conditions are given by equations (1) and (2) (derived in [1]), where $u(x,t)$ is the displacement of point x at the instant of time t.

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\rho}{E}\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\eta}{E}\frac{\partial u}{\partial t} = 0 \tag{1}$$

$$EA\frac{\partial u(x,t)}{\partial x}|_{x=0} = -F(t) \qquad , \qquad EA\frac{\partial u(x,t)}{\partial x}|_{x=L} = 0 \tag{2}$$

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By taking the Laplace Transform of equation (1), the solution on the complex frequency domain may be shown to be :

$$U(x,s) = \frac{F(s)cosh(k(x-L))}{EAksinh(kL)} \qquad , \qquad k = \sqrt{\frac{\rho s^2 + \eta s}{E}} \tag{3}$$

The RB mode can be removed by either expanding equation (3) as a spatial Fourier Series and removing the term that does not depend on x or by considering the ordinary differential equation of the RB motion of the rod. In either case, after removing de RB mode and making $s = i\omega$ it is possible to obtain the solution in the frequency domain given by equation (4). Taking the Inverse Fourier Transform of equation (4) (via IFFT) and adding it to the RB motion obtained separately from the Laplace solution yields the results shown on figure (1) (calculated for the point $x = 1$), which are plotted against the FEM solution for comparison.

$$\hat{u}(x,\omega) = \frac{-\hat{F}(\omega)cos(k(x-L))}{EAksen(kL)} + \frac{\hat{F}(\omega)}{EALk^2} \qquad , \qquad k = \sqrt{\frac{\omega^2\rho - i\omega\eta}{E}} \tag{4}$$

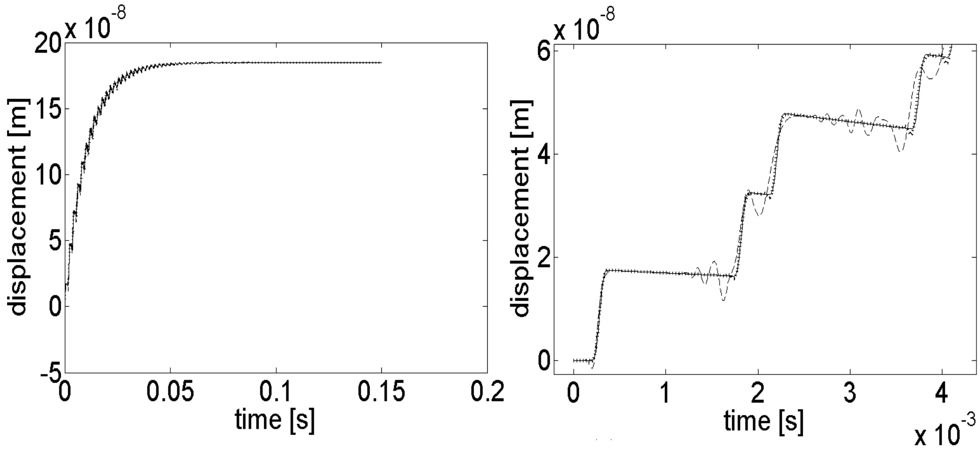

Figure 1: Displacement by spectral solution (–), FEM 20 (--) and 100 (··) elements

3 Conclusion and Acknowledgements

By comparing the results with the FEM solution it is possible to observe that by increasing the number of elements in the FEM mesh there is an attenuation of the *Gibbs Phenomenom* and an increasing convergence of the solutions.

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References

[1] James F. Doyle, *Wave Propagation in Structures.* Springer, New York, 1997.