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Chaotic pendulum excited by a magnetic force: An experimental approach

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This paper's goal is to model, simulate, build and instrument a physical pendulum, excited by a magnetic external force for chaos analysis. The chaotic pattern was defined through a phase portrait's analysis and the Maximum Lyapunov Exponent found beyond the visualization of the acceleration signal in the frequnecy domain given by the Fast Fourier Transform (FFT).

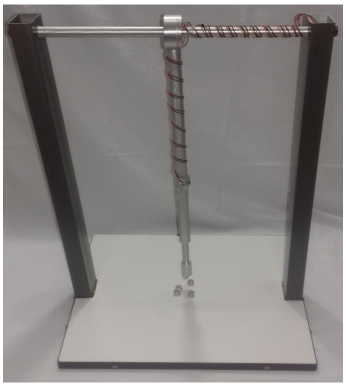

Figure 1: Built pendulum to ascertain the chaotic regime.

In order to model the pendulum's equation one applied Newton's Second Law for rotations, considering also the external forces that works in the system. Therefore the final equation that governs the movement of the studied pendulum is given by:

$$I\ddot{\Theta} = mgd\Theta + \frac{1}{2}c\Theta l^2 + \sum_{i=1}^{n} \frac{\alpha_i l\Theta}{\left[(x_i - l\theta)^2 + y_i^2 + z_i^2\right]^2} \tag{1}$$

On Equation 1, Θ represents the angular position of the pendulum's stem. $\dot{\Theta}$ and $\ddot{\Theta}$ give as an output the angular velocity and angular acceleration respectively. m is the pendulum's

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mass, g is the local gravity acceleration, d is the distance between the pendulum's center of mass and its rotation axis, c is a damping constant and l is the pendulum's lenght. The position of each magnet is given by an orderly trio, x, y and z and α is a constant that sets the intensity of the magnetic force.

The results obtained by an experimental data acquisition can be observed on Figure 2. This output is similar to the ones obtained by the computer simulation driven by Equation 1.

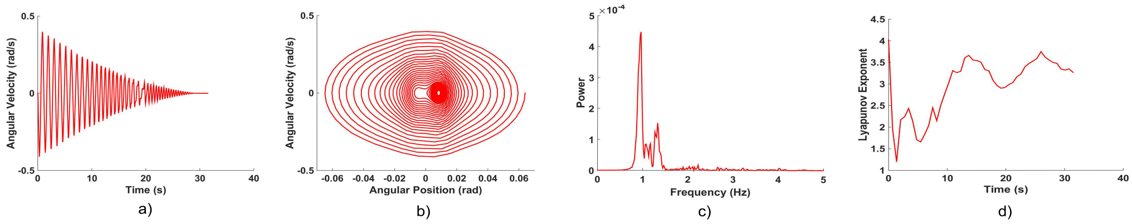

Figure 2: Experimental results, being: a) Time domain signal, b) Phase portrait, c) Frequency domain signal and d) The Maximal Lyapunov Exponent

In relation to the pendulum studied, the results were affirmative for the chaotic pattern according to both methods used - experimental data acquisition and computer simulation. The frequencies given by both charts were the system's natural frequencies. The numerical simulation's output was of 0.918 hz and the experimental result of 0.957 hz. Positive values were found for the Lyapunov Exponent as expected. The results found were: 2.345 according to the numerical simulation and 3.267 according to the experiments made.

For future papers a deeper analysis of the chaotic pattern of the given pendulum is intended to be achieved. Therefore bifurcation diagrams, Poincaré sections, the Wavelet transform and the Feigenbaum number must be included on the research.

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