Proceeding Series of the Brazilian Society of Applied and Computational Mathematics, Vol. 5, N. 1, 2017.

Trabalho apresentado no CNMAC, Gramado - RS, 2016.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

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Rigid bodies agglomeration simulation using ALE-FEM

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1 Introduction

This work aims the study of finite element methods discretized in dynamic meshes in order to simulate multiphase flow, particularly flows involving the agglomeration of dense packs of drops. The equations involved will be treated in an arbitrary lagrangian-eulerian framework (ALE) [1]. The numerical challenges range from the correct representation of interfaces, passing through the maintainability of the computer mesh, until the adequate representation of coalescence phenomena. We start considering rigid bodies surrounded by a newtonian fluid instead of drops [2, 3], situation that is modeled by the Navier-Stokes equations for the continuous phase, coupled with rigid bodies dynamics. This approach gives the possibility of analyzing rigid bodies collision processes, for the purpose of developping numerical coalescence simulation methodologies.

2 Numerical method

Let $\Omega(t) \subset \mathbb{R}^2$, $t \in [0, T)$, filled with an incompressible fluid of density ρ and viscosity μ, surrounding K rigid bodies $B_J(t)$ of mass M_J, $1 \leq J \leq K$, and

$$\begin{aligned} W_0(t) = \Big\{ (\mathbf{v}, \mathbf{Y}, \boldsymbol{\theta}) \,:\, & \mathbf{v} \in \left(H^1(\Omega(t))\right)^2, \mathbf{v} = \mathbf{0} \text{ on } \partial\Omega(t) \setminus \cup_{J=1}^{K} \partial B_J(t), \\ & \mathbf{Y} = (\mathbf{Y}_J)_{J=1}^{K}, \boldsymbol{\theta} = (\boldsymbol{\theta}_J)_{J=1}^{K}, \mathbf{Y}_J \in \mathbb{R}^2, \boldsymbol{\theta}_J \in \mathbb{R}^3, \\ & \mathbf{v}(\mathbf{x}, t) = \mathbf{Y}_J(t) + \boldsymbol{\theta}_J(t) \times (\mathbf{x} - \mathbf{x}_J(t)) \text{ on } \partial B_J(t), 1 \leq J \leq K \Big\} \end{aligned} \tag{1}$$

in which $\mathbf{x}_J(t)$ is the center of mass of $B_J(t)$. A variational formulation for this fluid-solid system, found in [2, 3], reads: Find $(\mathbf{u}, \mathbf{V}, \boldsymbol{\omega}) \in W_0(t)$, $p \in L^2(\Omega(t))/\mathbb{R}$, for almost all $t \in [0, T)$, such that,

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$$\rho \int_{\Omega(t)} \left[\delta_t \mathbf{u} + ((\mathbf{u} - \hat{\mathbf{u}}) \cdot \nabla)\, \mathbf{u}\right] \cdot \mathbf{v} + 2\mu \int_{\Omega(t)} D\left(\mathbf{u}\right) : D\left(\mathbf{v}\right) - \int_{\Omega(t)} p \nabla \cdot \mathbf{v}$$
$$+ \sum_{J=1}^{K} M_J \dot{\mathbf{V}}_J \cdot \mathbf{Y}_J + \sum_{J=1}^{K} \left(\mathbb{I}_J \dot{\boldsymbol{\omega}}_J + \boldsymbol{\omega}_J \times \mathbb{I}_J \boldsymbol{\omega}_J\right) \cdot \boldsymbol{\theta}_J = \rho \int_{\Omega(t)} \mathbf{g} \cdot \mathbf{v} + \sum_{J=1}^{K} M_J \mathbf{g} \cdot \mathbf{Y}_J, \quad (2)$$

for all $(\mathbf{v}, \mathbf{Y}, \boldsymbol{\theta}) \in W_0\left(t\right)$, and

$$\int_{\Omega(t)} q \nabla \cdot \mathbf{v} = 0, \quad (3)$$

for all $q \in L^2\left(\Omega\left(t\right)\right)/\mathbb{R}$, with

$$\begin{aligned} \mathbf{u} &= \mathbf{u}_D, \text{ on } \partial\Omega\left(t\right) \setminus \cup_{J=1}^{K} \partial B_J\left(t\right), \\ \mathbf{u}\left(\mathbf{x}, t\right) &= \mathbf{V}_J\left(t\right) + \boldsymbol{\omega}_J\left(t\right) \times \left(\mathbf{x} - \mathbf{x}_J\left(t\right)\right), \text{ on } \partial B_J\left(t\right), \\ \dot{\mathbf{x}}_J &= \mathbf{V}_J. \end{aligned} \quad (4)$$

Here, $\mathbf{u}$ is the fluid velocity, p the pressure, $\mathbf{V}_J$, $\boldsymbol{\omega}_J$ and $\mathbb{I}_J$ are the translational and angular velocities, and moment of inertia matrix of B_J, respectively. The evolution laws of the rigid bodies are transformed in natural conditions for the variational formulation through the surface forces acting over fluid-solid interfaces. The ALE derivative $\delta_t \mathbf{u}$ comes from considering an arbitrary referential coordinate with velocity $\hat{\mathbf{u}}$, which is the solution of an elasticity problem [4]. The finite element method (FEM) is used for spatial discretization, giving rise to a non-linear DAE system, resolved with *midpoint rule* and Adams-Bashforth like methods, and Newton-Raphson iterations.

3 Conclusions

A fluid-solid configuration allows us addressing the collision problem in order to create adequate coalescence models.

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