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Extraction of robust features on human faces

Amanda Figur[1]
ICMC/USP, São Carlos, SP
Antonio Castelo[2]
ICMC/USP, São Carlos, SP
Farid Tari[3]
ICMC/USP, São Carlos, SP

Robust features are points, curves or regions on a surface in $\mathbb{R}^3$ which capture important aspects of its geometry. The terminology *robust features* was introduced by Ian Porteous as they can be followed when the surface is deformed. They are used extensively in computer vision and shape recognition.

Regarding human faces, the nose ridge is an already known robust feature and is used in facial recognition methods [4]. A ridge is an example of a robust feature on a surface. It is the locus of points on the surface where one of the principal curvatures is extremal along its associated lines of principal curvature. It is, in general, a curve on the surface. Our big goal is to extract robust features (ridges and other special curves) on human faces in order to understand the geometry of faces, recognise faces and reconstruct them from the robust features. We observe that there is work on the extraction of ridges on human faces, see for example, ridges on Max Planck [7] and on David [2] of Michelangelo.

A survey on ridge extraction methods in [5] shows that the algorithm developed by Musuvathy *et al.* in [6] is still a state of the art method for ridge extraction on parameterized surfaces. In addition to the algorithm, the work in [6] indicates an important reference [3] for validating results. Cazals *et al.* gave in [3] an equation $P = 0$ that encodes the ridges and umbilic points (points where the two principal curvatures are equal) of a smooth parameterized surface.

As a first step towards extracting robust features on human faces, we implemented the algorithm in [3] using Maple 18. We extracted ridges (in the parameters space) of surface patches in $\mathbb{R}^3$ given as graphs of functions $w = h(u, v)$. The extracted ridges for the cases $h = h_i, i = 1, 2, 3$, with

$$h_1(u,v) = 7u^3 + 2u^2v + 3uv^2 + u^2 + 2v^2, \tag{1}$$

$$h_2(u,v) = -\frac{1}{12}u - \frac{35}{24}v + \frac{5}{2}uv + \frac{1}{2}u^2v - 3uv^2 - \frac{17}{12}u^2 + \frac{4}{3}v^2 + u^3 + \frac{1}{3}v^3 + \frac{5}{16}, \tag{2}$$

$$\begin{aligned} h_3(u,v) =& 116u^4v^4 - 200u^4v^3 - 312u^3v^4 + 108u^4v^2 + 592u^3v^3 + 252u^2v^4 - 24u^4v - 360u^3v^2 \\ & - 504u^2v^3 - 56uv^4 + 80u^3v + 324u^2v^2 + 112uv^3 - 72u^2v - 72uv^2 + 16uv, \end{aligned} \tag{3}$$

are given in Figure 1. Figure 1c shows some regions circled in red where our implementation of the algorithm in [3] runs into difficulties. These difficulties occur mainly at umbilic points. Our plots, using `implictplot` command in Maple 18 with the number of points option set to one hundred thousand (on a PC Intel i3 6th gen., with 12gb RAM and a HD), took five to ten minutes to execute. We believe that, using a more powerful computer, it would be possible to improve the

[1] amanda.figur@usp.br
[2] castelo@icmc.usp.br
[3] faridtari@icmc.usp.br

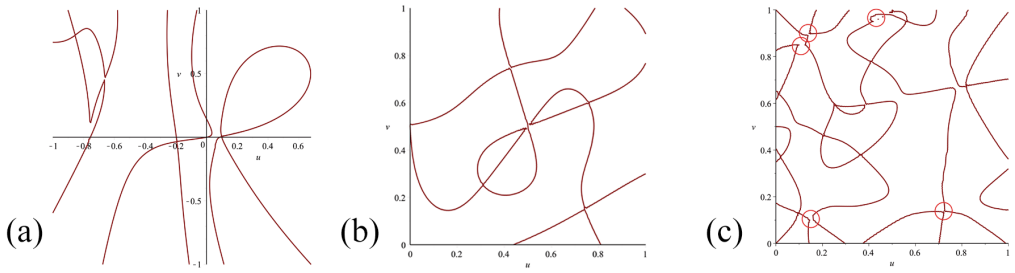

Figure 1: Ridges in the parameter space of surfaces extracted by implementing the algorithm in [3]: (a) for (1) with domain $[-1,1]\times[-1,1]$; (b) for (2) with domain $[0,1]\times[0,1]$; (c) for (3) with domain $[0,1]\times[0,1]$.

quality of our plots by setting the number of points option in `implictplot` to an even higher number of points.

The aim of our work is to implement the algorithm described in [6] in order to extract ridges on human faces. The algorithm has greater accuracy when the surfaces are parameterized by B-Splines. We aim to extract ridges on the dataset [1] and assemble a database of robust features of faces. We hope that this will lead to a better understanding of the geometry of human faces.

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