Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 8, n. 1, 2021.

Trabalho apresentado no XL CNMAC, Evento Virtual - Co-organizado pela Universidade do Mato Grosso do Sul (UFMS).

Variable density band-based undersampling scheme for Compressed Sensing MRI reconstruction

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Magnetic resonance of brain images have an implicit sparsity in an appropriate transform domain [1]. Based on compressed sensing theory (CS), images with a sparse representation can be recovered from undersampled k-space data. However, to have an efficient reconstruction [1], the undersampling scheme should be incoherent with respect to the sparsifying transform [2].

Several strategies for incoherent undersampling can be encountered in the literature (see [1] and reference therein). For instance, random uniform undersampling has been proven to offer correct incoherent properties. Randomness shows both mathematical simplicity and guarantees the near-optimal degree of incoherence [2]. However, fully random 2D sampling is not feasible in terms of hardware. To deal with this problem, the lines of a Cartesian trajectory to fully sample the k-space can be selected uniformly at random, i.e., the k-space is randomly undersampled only in one Cartesian direction [2].

In general, the k-space representation (for instance Fourier coefficients) of MRI images does not follow a uniform distribution, i.e., most of the energy in the k-space is concentrated near the origin, where the lowest, most relevant spatial frequency information is located [2, 3, 6]. This suggests that in a real scenario, undersampling should be denser in the central region of k-space and then accordingly be diminished following a variable density scheme [1]. As a result, one should build a sampling scheme that follows an exponential probability density function (PDF), where the higher probability of drawing a particular sample resides in the center of k-space.

In this work, we consider a combination of a Cartesian variable density and a random uniform undersampling scheme. To this end, we divided the entire region in n frequency bands, whose size or sampling density has varied according to a normalized, discrete exponential PDF:

$$PDF(k) = \frac{c(k)}{\sum_{k=1}^{n} c(k)}, \tag{1}$$

where $k \in \{1, 2, ..., n\}$ is the number of each band in the grid, $c(k) = k^v$, v is the degree of the exponential function, and for our numerical experiments we let v vary in the range [1,4]. We proposed an undersampling scheme generator that incorporates an exponential decay combined

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with a degree of randomness, based on two modes: (a) varying the number of samples within the bands (Mode 0), and (b) varying the width of the band (Mode 1), where every line in the Cartesian direction have the same probability of being sampled.

Our proposal allows the undersampling scheme to exhibit a higher incoherence measure in accordance with the Transformed Point Spread Function (TPSF) metric in comparison to other variable density schemes (polynomial, exponential, or schemes based on the golden ratio). In order to obtain the undersampled solution in the k-space, an associated optimization problem in (2) is formulated:

$$min\big(\parallel \mathcal{F}_s y - x \parallel_2^2 + \lambda \parallel \Psi y \parallel_1 \big) \tag{2}$$

where F_s, y, x, and Ψ represent the Fourier operator including the proposal undersampling scheme, the reconstructed real image, the undersampled k-space input, and the wavelet operator, respectively.

The proposed approach for undersampling was tested in brain image reconstruction and compared with results presented in [1]. The problem in equation (2) was solved with a non-linear conjugate gradient algorithm included in the library *SparseMRIV0.2*© (publicly available in following link: `https://people.eecs.berkeley.edu/~mlustig/Software.html`). Incoherence (TPSF), mean square error (MSE), peak signal to noise ratio (PSNR), and structural similarity index measure (SSIM) reconstruction metrics have been used. The numerical results with tested images (from [1]) of the proposed undersampling approach show an improvement up to 19% in TPSF, 71% in MSE, 20% in PSNR, and a similar value in SSIM, which means that the reconstructed image is better than the reference obtained at the literature.

Ackowledgments

CES acknowledges the FEEI-PROCIENCIA-CONACYT: PRONII. Authors acknowledge the support given by the group of research CC&MA.

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