Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 8, n. 1, 2021.

Trabalho apresentado no XL CNMAC, Evento Virtual - Co-organizado pela Universidade do Mato Grosso do Sul (UFMS).

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Symmetric circular formations of mobile agents under time-delays and phase-lags

Vander L. S. Freitas
Department of Computing, Federal University of Ouro Preto, Brazil

Serhiy Yanchuk
Institute of Mathematics, Technical University of Berlin, Germany

Helder L. C. Grande
National Institute for Space Research, Brazil

Elbert E. N. Macau
Federal University of Sao Paulo, Brazil

The control of multiple mobile agents has applications in agriculture, data collection, and space engineering. We investigate the effects of time-delay (τ) and symmetrically assigned phase lags (ψ) in a first-order model of particles with coupled-oscillator dynamics aiming at circular symmetric formations under all-to-all interaction [1, 2]. For $\tau = 0$ and $\psi = 0$, the standard dynamics is composed of agents developing circular trajectories and grouping into M clusters, with the same number of participants, symmetrically positioned along the trajectory [3], with M being a parameter.

We first investigate the system without phase lags. The time-delay represents either an internal processing time or the signal propagation time between agents. For small time-delay ($\tau < \tau_c$), the formations remain the same as for $\tau = 0$, and the larger the M is the smaller τ_c becomes. This means that a system with only a few clusters is more robust to time-delay than another with many clusters.

Above the threshold τ_c, the clusters reappear stably when τ comes close to the resonance with the agents rotation period. Between the resonances, there are different symmetric configurations with the number of clusters less than or equal to M.

The second part of the work considers already formed clusters as the initial condition. We then add a phase lag (ψ) to the dynamics of agents within the same cluster and another phase lag with the opposite sign ($-\psi$) to the neighboring clusters. Tests are conducted with configurations of even clusters to ensure that the same number of agents receive ψ and $-\psi$. Depending on the intensity $|\psi|$, either incoherent or alternative symmetric arrangements emerge. The incoherent part is characterized by intermediate values of the Kuramoto m-th order parameters, neither close to zero nor one. The particles move in circles, but without any organized pattern. On the other hand, the alternative symmetric arrangements correspond to other symmetric cluster formations or clusters with different spacing between each other. This last possibility is not observed in the original system or with the time-delays alone.

The resulting formations are sometimes classified as incoherent, but when one evaluates the order parameters of each of the two groups (the agents with ψ and those with $-\psi$) separately, they sometimes present well-defined clusters themselves.

In summary, the time-delay and phase lags play the role of effective control parameters that allow to create new stable formations of mobile agents with various degrees of symmetry and coherence.

Acknowledgments

S.Y. was funded by the "Deutsche Forschungsgemeinschaft" (DFG) in the framework of the project 411803875. H.L.C.G. was funded by the research scholarship PCI-INPE, process 301113/2020-3. E.E.N.M. and V.L.S.F. would like to thank grants 2015/50122-0 and 2018/03211-6, São Paulo Research Foundation (FAPESP), AWS/CNPq grant 440034/2020-5, and CAPES grant 23038.014333/2020-46. V.L.S.F. also thank the Federal University of Ouro Preto.

References

[1] V. L. S. Freitas, S. Yanchuk, M. Zaks, E. E. N. Macau. Synchronization-based symmetric circular formations of mobile agents and the generation of chaotic trajectories, *Communications in Nonlinear Science and Numerical Simulation*, 94: 105543, 2021. https://doi.org/10.1016/j.cnsns.2020.105543.

[2] R. Sepulchre, D. A. Paley, and N. E. Leonard. Stabilization of Planar Collective Motion: All-to-All Communication, *IEEE Transactions On Automatic Control* 52 (5): 811-24, 2007.

[3] V. L. S. Freitas, S. Yanchuk, H. L. C. Grande, E. E. N. Macau. The effects of time-delay and phase lags on symmetric circular formations of mobile agents, *European Physical Journal Special Topics*, 2021. To appear.

[4] S. Yanchuk, P. Perlikowski. Delay and periodicity. *Physical Review E* 79 (4): 046221, 2009. 10.1103/PhysRevE.79.046221