

RLC circuit: Analogy with mechanical systems.

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Introduction

In studies of mechanical systems in general, the need to understand how a mechanical system works, in addition to how it behaves, is essential. In this paper, the comparison between an RLC circuit with a Mass-Spring-Damper (MSD) system is done through python programming, using the numpy and scipy.integrate libraries. The circuit will be presented, as well as its responses in the time domain and in the frequency spectrum, in addition to a brief comparison between an electrical circuit and a mass-spring-damper.

The RLC system equation

The RLC circuit, formed by a resistor, an inductor and a capacitor, connected in series, has the equation formed through the Kirchhoff's law, or Kirchhoff's loop rule, and has the following form:

$$\frac{d^2 q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{1}{LC} q = \frac{E}{L} \quad (1)$$

$$E = E_0 \text{sen}(wt) \quad (2)$$

Where q is the electric charge, R the resistance, L the inductor and E is the source that varies the voltage over time. The variable source used in the experiment has the behavior following the equation 2. Below, a diagram with the representation of the RLC circuit and Mass-Spring-Damper.

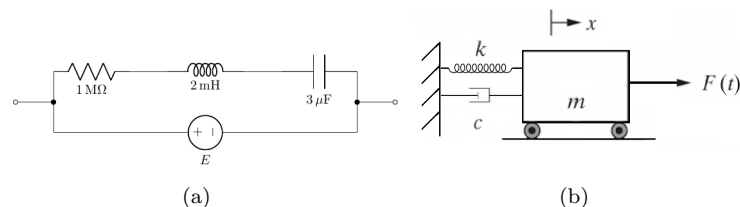


Figure 1: a)RLC Circuit, b)Mass-Spring-Damper

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Results

Using as parameters for $R = 2 \Omega$, $C = 1/120 \text{ F}$, $L = 1 \text{ H}$, $q = 1 \text{ C}$ and the initial voltage and angular frequency of the variable source $E_0 = 10 \text{ V}$ and $\omega = 8 \text{ rad/s}$, in a time of 10s, the results of Figure 2a are obtained. The figure 2b shows a direct comparison between the results of the Electric Current of the RLC circuit with the speed of the MSD.

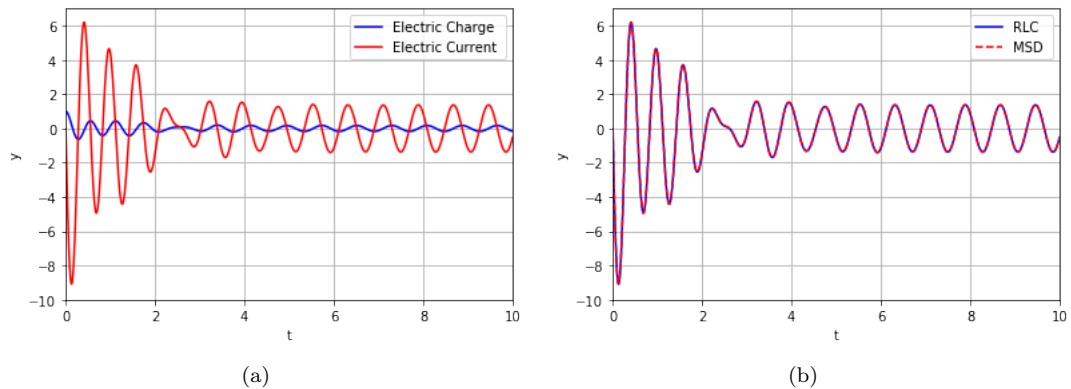


Figure 2: a)RLC Circuit, b)RLC X MSD

The mass-spring-damper system used in the comparison was simulated in python with the following parameters: viscous damping constant 'c' with value 2 (Ns / m), spring constant 'k' equal to 120 (N / m), value of mass 'm' equal to 1 (kg) and applied with an external force.

Figure 2b makes it clear that the behavior of the MSD system is the same as the behavior of the RLC system. Thus, it can be said that the Electric Charge (q) is analogous to displacement (x), the Electric Current (dq / dt) to Speed (dx / dt), the Voltage of the variable source $E (t)$ to the External Force $F (t)$, the Inductance (L) to the mass (m), the Resistor (R) to the Viscous Damping Coefficient (c) and the Capacitance (C) to the Spring Elasticity Constant (K).

Conclusion

When analyzing the simulated results, it is observed that the RLC circuit has a completely linear behavior. In addition, a observed characteristic was the drop, with quadratic behavior, of the values on the y axis (charge and electric current), and at the end its steady state. As it is a system with variable external force, the loss of energy due to the capacitor charging and the resistances of the circuit, right when the circuit is turned on, are compensated through the variable source, making the oscillations of the current and electric load values predictable from 4s.

References

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