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Remarks on the Dynamics of the Double Pendulum: A Frequency Domain Approach.

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Introduction:

The present paper aims to present a dynamic analysis of a double pendulum (Figure 1) in the time and frequency domain, using python and its numerical libraries numpy and scipy [1]. Simple physical model based on two pendulums, one being fixed at the other's end, strongly nonlinear and sensitive to initial conditions that can present a chaotic response [2].

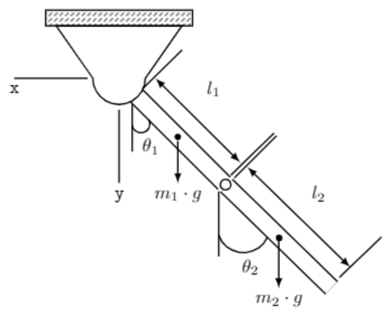

Figure 1: Free body diagram

Equation of motion:

The equation of motion of this system is found using the Lagrange method, this method being based on the kinetic and potential energy of the system.

The equation of motion of the double pendulum:

$$L = \frac{1}{2}(M_1+M_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}M_2 l_2^2\dot{\theta}_2^2 + M_2 l_1 l_2 \dot{\theta}_1\dot{\theta}_2 \cos(\theta_1-\theta_2) + (M_1+M_2)g l_1 \cos(\theta_1) + M_2 g l_2 \cos(\theta_2) \tag{1}$$

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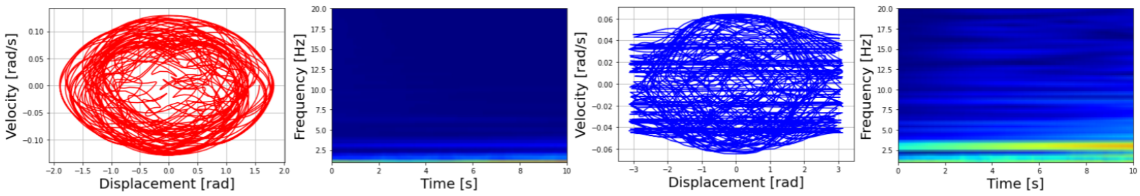
Assuming for this system $l_1 = l_2 = l$ and $M_1 = M_2 = M$ and rearranging the ode obtained of the eq.1 as a system of first order diferential equations, for the numerical solution the following variables are introduced $P_{\theta 1} = \dot{\theta}_1$ and $P_{\theta 2} = \dot{\theta}_2$ therefore the equations becomes:

$$\begin{cases} \dot{\theta}_1 = \frac{6}{Ml^2} \frac{2P_{\theta 1} - 3\cos(\theta_1 - \theta_2) P_{\theta 2}}{16 - 9\cos^2(\theta_1 - \theta_2)} \\ \dot{\theta}_2 = \frac{6}{Ml^2} \frac{8P_{\theta 2} - 3\cos(\theta_1 - \theta_2) P_{\theta 1}}{16 - 9\cos^2(\theta_1 - \theta_2)} \\ \dot{P}_{\theta 1} = -\frac{1}{2} M l^2 [\dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + 3\frac{g}{l}\sin(\theta_1)] \\ \dot{P}_{\theta 2} = -\frac{1}{2} M l^2 [-\dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + \frac{g}{l}\sin(\theta_2)] \end{cases} \tag{2}$$

Discussion of results

By analyzing the phase portrait responses (Figure 2 a, Figure 2 c) and the Wavelet transform (Figure 2 b, Figure 2 d), two of the tools which are often used to detect chaos, the behavior of the system is observed. The simulation results present abnormal behavior, because when analyzing the phase portrait of periodic systems (systems with simple behavior) it is observed that the lines do not cross frequently, generating a response similar to an ellipse, but in the case of Figure 2 and Figure 2 c it is observed that this phenomenon does not occur. Moreover, in the Wavelet transform of Figure 2 b and Figure 2 d it is not possible to count the frequencies, unlike periodic systems, which is a high indication of chaos, besides being a system sensitive to initial conditions where the slightest variation can drastically alter the response of the system.



(a) (b) (c) (d)

Figure 2: a)Phase portrait θ_1, b)CWT θ_1, c)Phase portrait θ_2, d)CWT θ_2

Conclusion

Although the double pendulum is a relatively simple mechanical system, it can exhibit chaotic response if they exhibit abnormal behavior as described, which is observable using tools such as phase portrait and wavelet transform. To perform such tasks, python proves to be a very competent and relatively simple tool to use.

References

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