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Truncated Conjugate Gradient Method for History Matching in Reservoir Simulation

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Abstract. History Matching (HM) is an important problem in Oil Reservoir Simulation. We present here the Truncated Conjugate Gradient (TCG) Method to solve this problem. We compare TCG with two other well known schemes, TSVD and L-BFGS, in a numerical experiment using the benchmark problem PUNQ-S3. Our results indicate that TCG is a valuable tool for HM.

key-words Oil Reservoir Simulation, History Matching, Truncated Conjugate Gradient.

1 Introduction

Simulations of multiphase flows in porous media are the “nuts and bolts” of petroleum-reservoir engineers. A reservoir simulator is the piece of software that computes, for instance, the evolution of oil and gas production rates or bottom-hole pressure at each well, by means of numerically solving the appropriate partial differential equations. See [9].

A reservoir simulator takes as input a model of the reservoir: a description of its geometry, of the properties of the rock and of the fluids, etc. Such data are not readily available for natural reservoirs (often inconveniently buried under miles of water, rock and salt), but rather obtained indirectly, from seismic data or geological knowledge about the formation. Using this *a priori* information, an exploitation strategy for the reservoir is conceived and implemented. After some years, the history of the field (past production data at wells) is often quite distant from what had been predicted by simulations based on the prior information.

History Matching (HM) is a data-assimilation technique, in which the history is used in order to improve the quality of the reservoir characterization. The basic idea is to look for reservoir parameters (such as permeability or porosity) for which simulation better matches the historical data actually recorded.

Various schemes have been designed to solve the HM problem, such as L-BFGS [3, 13] and TSVD [1, 10, 12]. Most recently, we have proposed the Truncated Conjugate Gradient

(TCG) algorithm for HM [2]. As we describe in this work, TCG is based on the Gauss-Newton method. At each step, the optimization problem is linearized around the current approximation and the corresponding linear system is solved using the Conjugate Gradient method. As is usually the case with ill-posed problems, some regularization must be introduced. Here, this is done through a truncation procedure. We compare the behavior of TCG, TSVD and L-BFGS in some numerical experiments. In our results, TCG clearly outperforms TSVD. TCG displays a higher convergence rate than L-BFGS, although the latter runs faster.

The rest of this paper is organized as follows. In Section 2 we present a Bayesian formulation for the HM problem. In Section 3 the TCG algorithm is discussed. We recall the formulations of L-BFGS and TSVD in Section 4. Section 5 portrays a comparative study of the behavior of TCG, L-BFGS, TSVD in the PUNQ-S3 reservoir model [6]. This is a well known benchmark problem in HM.

2 Mathematical Model

Following [8], we assume that the *a priori* model is a Gaussian random variable whose probability distribution function (pdf) is given by

$$f_{prior}(m) \propto \exp\left(-\frac{1}{2}\|m - m_{prior}\|_{C_M^{-1}}^2\right).$$

Here, m_{prior} is the *a priori* mean, C_M is the *a priori* covariance matrix and, for a symmetric positive-definite matrix A , $\|\cdot\|_A$ stands for the norm $\|m\|_A = \langle Am, m \rangle^{1/2}$.

Usually, the *a priori* information is rather poor. Therefore, the true reservoir production is often far from the prediction. In History Matching, one uses this new information, the actual history of the reservoir production (oil, gas and water productions, bottom-hole pressures at the wells, etc.) to better characterize the reservoir.

We consider that the vector ϵ of measurement errors of the history data d_{obs} is a normal random variable with covariance C_D . Further assuming that ϵ is independent of the *a priori* model, the *a posteriori* distribution (conditioned to the history data) satisfies

$$f_{m|d_{obs}}(m) \propto \exp\left(-\frac{1}{2}(\|m - m_{prior}\|_{C_M^{-1}}^2 + \|g(m) - d_{obs}\|_{C_D^{-1}}^2)\right).$$

Here $g(m)$ is the vector of observations associated with model m (i.e., g represents the reservoir simulator) and d_{obs} is the vector of the actual observations, see [8]. One would like to generate a set of reservoir models sampling $f_{m|d_{obs}}$. This is, in general, too costly. A less expensive procedure is to obtain the Maximum a Posteriori (MAP) solution

$$m_{map} = m_{map}(m_{prior}, d_{obs}) = \arg \min_m J(m), \quad (1)$$

where

$$J(m) = \|m - m_{prior}\|_{C_M^{-1}}^2 + \|g(m) - d_{obs}\|_{C_D^{-1}}^2. \quad (2)$$

A set of solutions to the problem can be obtained from the Randomized Maximum Likelihood (RML) strategy [4,8]. RML amounts to generating a set of n vectors of observations $\{d_{obs}^{(i)}\}_{i=1,\dots,n}$ and of models $\{m_{prior}^{(i)}\}_{i=1,\dots,n}$ according to the respective probability distributions and to obtain MAP solutions $m_{map}^{(i)} = m_{map}(m_{prior}^{(i)}, d_{obs}^{(i)})$ of (1). In this way, the use of RML reinforces the need of solving the optimization problem (1) efficiently.

As is usual with inverse problems, HM is ill-conditioned. A regularizing procedure is to consider the Cholesky decomposition $C_M = LL^T$ and the change of variables

$$\theta(m) = L^{-1}(m - m_{prior}), \quad m(\theta) = m_{prior} + L\theta, \tag{3}$$

so that $\|m - m_{prior}\|_{C_M^{-1}} = \|\theta\|$. The MAP problem in θ is to obtain θ_{map} such that

$$\theta_{map} = \underset{\theta}{arg \min} J^\theta(\theta), \tag{4}$$

where $J^\theta(\theta(m)) = J(m)$, i.e., $J^\theta(\theta) = \|\theta\|^2 + \|g(m(\theta)) - d_{obs}\|_{C_D^{-1}}^2$.

3 TCG

We now describe briefly the Truncated Conjugate Gradient (TCG) scheme to solve (4). More details can be found in [2]. We use Gauss-Newton, so that at each step $g(m(\theta))$ is replaced by its linearization around the current estimate θ_k , i.e., $g(m(\theta)) \approx g_k + G_k L \Delta\theta$, where $g_k = g(m_k)$, $m_k = m(\theta_k)$, $\Delta\theta = \theta - \theta_k$ and $G_k = (d/dm)g(m_k)$, the derivative of the observations with respect to the model parameters, is the so-called sensitivity matrix. We thus obtain the linear least squares problem

$$\min_{\Delta\theta} J_k^\theta(\Delta\theta), \tag{5}$$

where $J_k^\theta(\Delta\theta) = \|\theta_k + \Delta\theta\|^2 + \|G_k L \Delta\theta - r_k\|_{C_D^{-1}}^2$, with $r_k = d_{obs} - g_k$. Then (5) is equivalent to solving

$$(I + L^T G_k^T C_D^{-1} G_k L) \Delta\theta_k = L^T G_k^T C_D^{-1} r_k - \theta_k. \tag{6}$$

Notice that $L^T G_k^T C_D^{-1} r_k - \theta_k = -\nabla J_k^\theta/2$ and $I + L^T G_k^T C_D^{-1} G_k L = H_k/2$, where H_k is the Hessian of J_k^θ . We apply the standard conjugate gradient (CG) algorithm (see, e.g., [5]) to solve (6). However, similarly to what happens with the TSVD algorithm, one has to appropriately truncate the CG iteration. Finally, the solution obtained using TCG provides a direction in which a line search is performed. It is possible to implement TCG without having to compute the Cholesky decomposition of C_M . This is relevant for large-scale problems, since the factorization cost scales with the cube of the grid size.

4 TVSD and L-BFGS

We briefly describe TSVD and L-BFGS, which have been previously employed in HM, see [1, 3, 10, 12, 13].

Analogously to TCG, TSVD is also based on Gauss-Newton. As with TCG, at step k one has to solve (6). The solution can be obtained as a linear combination of the right singular values of $C_D^{-\frac{1}{2}}G_kL$. In TSVD, the summation in this linear combination is truncated in an appropriate way, see [1, 10, 12] for further details.

The BFGS iterative method to solve the general problem of minimizing a function $f(x)$ was proposed simultaneously in 1970 by Broyden, Fletcher, Goldfarb and Shanno. The central idea is to construct at step k a search direction Δx_k using a rank-two update of the approximation H_k of $f''(x_k)$. The limited-memory L-BFGS(n) scheme uses only the information of the n most recent steps to build H_k . For more details, see [4, 7].

5 Numerical Experiments

We present numerical experiments comparing the performance of TCG, TSVD and L-BFGS for the HM problem. We observe that at each iteration TCG and TSVD approximately solve (6). This resolution involves the application of both the sensitive matrix G_k and of its adjoint G_k^T to vectors. In L-BFGS, the approximation of the Hessian is based solely on gradient values at previous iterates and therefore only applications of G_k^T are required. In our experiments, we make use of SIMPAR, a basic fully implicit 3D black-oil simulator developed by Petrobras and featuring tools for the computation of derivatives and adjoints.

We considered the benchmark PUNQ-S3 [6] for our tests. It is a model with a grid size of $19 \times 28 \times 5$ and 1,761 active blocks. Figure 1 depicts a colormap of the depth of the reservoir top. The initial oil-water contact (blue contour) and gas-oil contact (red contour), a sealing fault (black line) and six production wells (black dots) are also indicated. There is an aquifer to the West and to the North of the reservoir and there are no injector wells.

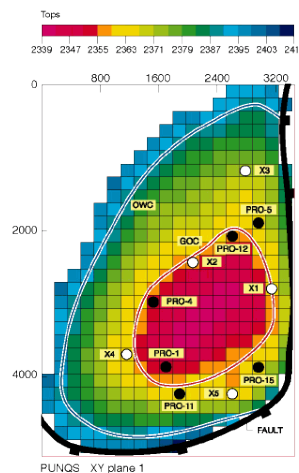


Figure 1: Colormap of the depth of the reservoir top, PUNQ-S3.

In this HM problem, the vector of parameters m consists of porosity and horizontal and vertical log-permeabilities at each grid cell. Geostatistical information about the parameters is given (basically, C_M), as well as measured values for these properties at well gridblocks (so-called hard data). This leaves us with about 5,200 parameters.

The history, consisting of bottom-hole pressure, water cut and gas-oil ratio at the wells, includes one year of extended well testing, followed by three years of field shut-in and four years of field production. The oil production rates for each well are given and used as boundary conditions, but if the minimum bottom-hole pressure for a well is reached, the simulation proceeds with such pressure prescribed.

Our PUNQ model displays some minor changes with respect to the original (see [2]) to accommodate for SIMPAR limitations. In our algorithm, we actually introduce a change of variables for the porosity, in order to generate an unconstrained optimization problem.

We have implemented RML to obtain a set of parameter fields honoring the geological *a priori* model, the hard data and the production history. Figure 2 shows the porosity fields of the true reservoir and of one solution obtained with TCG. They are quite different, as expected, since the HM problem has many solutions within the acceptable margin of error.

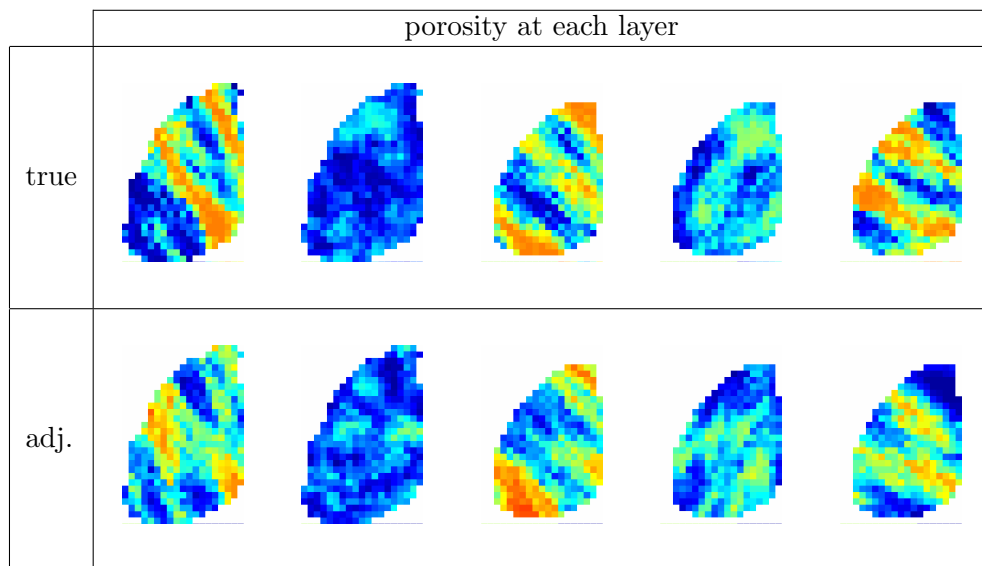


Figure 2: Porosity fields of the true and of one of the adjusted models.

We tested the ability of RML to predict the reservoir production. Figure 3 displays the graphs of the field cumulative productions of oil (left), gas (center) and water (right). Each green line represents the simulated production of one reservoir model obtained by use of TCG. To the left of the red line we have the 8-year history period, and to the right, the forecast for the following 8.5 years. The results for TSVD and L-BFGS are similar. We see that the true future behavior is contained in the fan of the extrapolations coming from RML. In those experiments, we consider that a simulation run has converged if $J(m_{final}) \leq D$. The value of D we choose comes from probabilistic considerations,

where the MAP solution is supposed to be a χ^2 random variable, see [8,11].

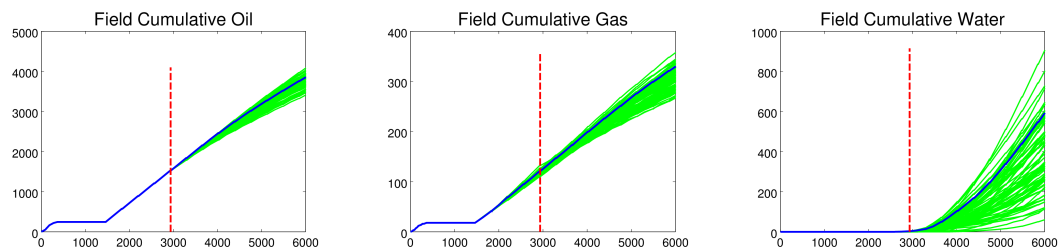


Figure 3: In green, extrapolations of cumulative oil, gas and water production. In blue, the true field. The red vertical line indicates the end of the history period.

Table 1 presents the total simulation time and the number of converged cases of TCG, TSVD and L-BFGS for 100 simulations using RML. TCG (n) indicates that at most n directions are used in each CG iteration. We see that TSVD had a poor behavior when compared with the other schemes. While TCG was the most robust scheme, with the greatest number of converged runs, L-BFGS was the least time consuming. The total time divided by the number of successes is comparable for these two schemes. For others experiments in which TCG had clearly the best overall performance, see [2].

Table 1: Converged cases and time for 100 simulations.

Algorithm	Converged cases	Total Time (sec.)
L-BFGS	35	33,843
TSVD	46	209,860
TCG (5)	79	73,254
TCG (10)	78	99,725

6 Conclusion

In this work we describe the TCG algorithm for History Matching. We used the standard benchmark PUNQ-S3 to compare TCG with TSVD and L-BFGS, two well known schemes for HM. In these experiments, TCG presented the most robust behavior, while L-BFGS displayed the smallest running time. We will develop further comparative studies on these schemes.

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