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People as planets: a gravity model for inter-municipal mobility

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Newton's law of gravitation states that any particle of matter in the universe attracts each other with a force that varies with their masses and the distance between them, such that $F_g = Gm_1m_2/d^2$. Centuries later, this idea was extended [2] to allow modeling not only the gravitational attraction between planets but also human mobility "attraction" between regions.

The gravity model has been a widely used and successful empirical model in the fields of economics [1]. Also, it has recently shown great potential in human mobility-related applications, such as the relationship between people's mobility and COVID-19 cases trends in China [4].

Gravity-based spatial interaction models can capture the absolute and relative spatial relationship of origins and destinations of travels in a region. The magnitude of these interactions relies on the population's sizes, the distance between them, and socioeconomic factors, such as job and study offer.

The state of Bahia has 28 Health Regions - a continuous geographical space constituted of neighbor municipalities and grouped by culture, economic, social, and infrastructural identities. In the present work, we optimize a gravity model [6] considering four different geographical scales, which are: Brazil as a whole, the North East region of Brazil, Bahia State, and Bahia's Health Regions.

The flow data used to train the model was obtained through "Road and waterway connections 2016" and contains the weekly travel frequency (flow) of vehicles between pairs of Brazil's cities. The frequencies are aggregated within the round trip - the flow from city A to city B is the same as from B to A. Moreover, as a distance metric, the Manhattan distance between each pair of cities (i.e., the roads paths distance) was used instead of the euclidean one.

The flow between a pair of regions i and j is given by $F_{ij} = KP_iP_j/d_{ij}^{\sigma}$, where P is the population of each region, d is the distance (in Kilometers) between them (Manhattan distance), and K and σ are the parameters optimized through the minimization of the objective function $S = \sqrt{(\sum F'_{ij} - F_{ij})^2/N}$, given that F'_{ij} is the real IBGE flow. Besides taking different scales into account, we use two different approaches for the optimization. Approach 1 considers all of the flow data for the training, and approach 2 considers only the maximum flow values in each of n equal distance intervals.

We found that when using approach 1 the model has high explanatory power when considering the Health Regions scale (Fig.1)($R^2 = 0.92$). However, as the scale gets smaller, so does the

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predictive power of the model. That is shown by the model's performance for Bahia state ($R^2 = 0.40$), North East region ($R^2 = 0.30$), and Brazil (R^2=0.27). On the other hand, with approach 2 (i.e. using only maximum flows), the model's performance on the training data was significantly higher regardless of the scale ($R^2 > 0.9$).

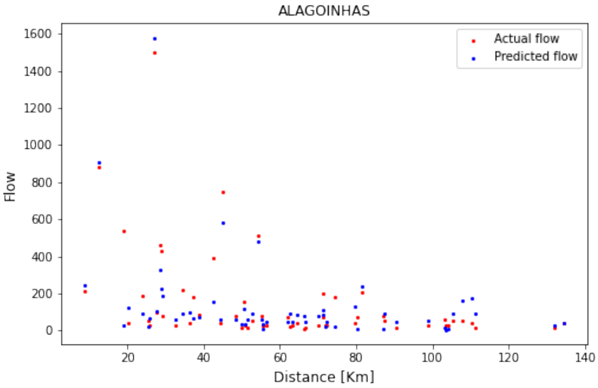

Figure 1: Gravity model optimization for Alagoinhas Health Region, with $K = 1.49x10^-5$ and $\sigma = 1.34$

One of the possible reasons for this is that when considering smaller scales, other factors beyond population and distance such as social-economical indicators and the city's attractiveness to outsiders impact significantly on mobility [7], and as they are not taken into account, it increases the model's bias.

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