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On a general model of rumor transmission

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1 Introduction

Spreading processes take place in many aspects of life. An example where these processes appear is in the study of the propagation of an infectious disease that could be mathematically modeled with the Susceptible-Infected-Removed epidemic model (SIR model). Other example, which has some similarity with the previous one, is the dissemination of an information on a population that could be investigated by mean of the Maki-Thompson model. These are compartmental models, for which is considered a population subdivided into different classes of individuals. This classes are called susceptible, infected and removed, for the SIR model; and ignorants, spreaders and stiflers for the rumor model. Both models can be seen as a system of three non-linear ordinary differential equations which may be solved numerically and allow us to understand the evolution of the classes along time. In this work we consider the approach in which both models arise as a particular case of a general rumor model. First, we discuss the case in which the population is homogeneously mixed; i.e., is represented by the complete graph. Then, we extend our discussion to show how heterogeneity in the interactions of the population could affect the respective mathematical analysis.

2 Motivation: homogeneously mixed populations

The general rumor model is the following system of Ordinary Differential Equations (ODE):

$$\left\{\begin{array}{lcl} \dfrac{dx}{dt} & = & -\lambda\, x(t)\, y(t), \\ \dfrac{dy}{dt} & = & \lambda\, x(t)\, y(t) - \delta\, y(t) - \alpha\, y(t)\,(1 - x(t)), \\ \dfrac{dz}{dt} & = & \delta\, y(t) + \alpha\, y(t)(1 - x(t)), \end{array}\right. \tag{1}$$

where $\lambda > 0, \alpha \geq 0$ and $\delta \geq 0$ are given constants, and $x(t)$, $y(t)$ and $z(t)$ denote, respectively, the proportion of ignorants, spreaders and stiflers at time t, $t \geq 0$. We let $x(0) = 1$ and $y(0) = z(0) = 0$ and we note that $x(t) + y(t) + z(t) = 1$ for any $t \geq 0$. The last assumption means that

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the population is closed. The SIR and the Maki-Thompson models are obtained by considering $\alpha = 0$ or $\alpha = 1$, respectively. See Figure 1 for an illustration of the evolution of these proportions.

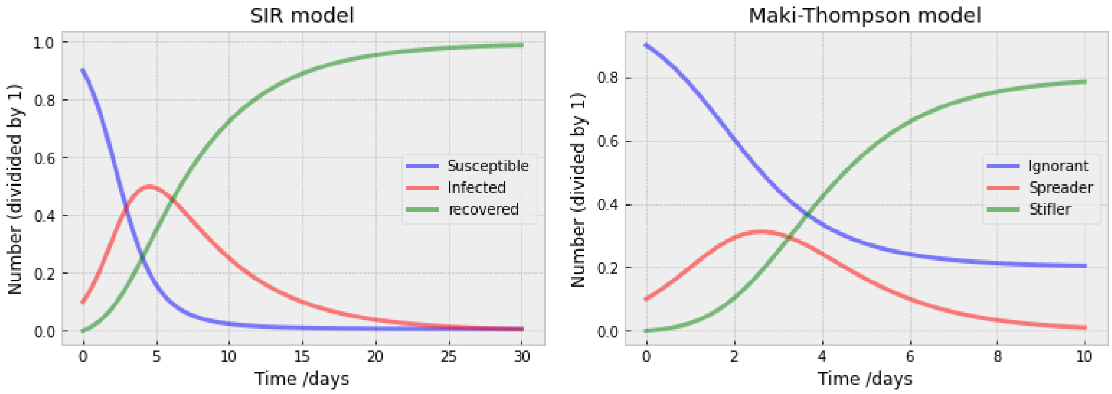

Figura 1: Numerical results with the dispersion model in (1).

We can formulate an stochastic version of the rumor model by considering the continuous-time Markov chain $\{(X_t, Y_t, Z_t)\}_{t\geq 0}$, with transitions and rates given by

$$\begin{array}{cc} \text{Transition} & \text{Rate} \\ (-1,1,0) & \lambda\, X\, Y, \\ (0,-1,1) & \alpha\, Y\,(N-1-X) + \delta\, Y. \end{array} \tag{2}$$

The random variables X_t, Y_t, Z_t denote, respectively, the number of ignorants, spreaders and stiflers at time t, for $t \geq 0$. The solution of (1) may be seen as a good approximation of the evolution of the proportions of ignorants, spreaders, and stiflers, for $t > 0$ and a large enough population, in the stochastic model. This justifies the fact that many results may be obtained directly from a suitable analysis of (1).

3 Our analysis

In this work we explore the connection between the deterministic and the stochastic version of the general rumor model and we extend it to some classes of finite graphs. In order to do it we discuss some theoretical properties of these models and perform computational simulations to understand the behavior of the model in some graphs. We follow the analysis of [1–3].

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