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Preserving properties of necessity and possibility connectives over dual and conjugate triangular norms

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1 Introduction

Fuzzy set theory introduced by Zadeh in 1965 has been applied to many fields [3]. Due to the increase in the complexity of real problems mainly related to insufficient knowledge of the problem domain, there exist some uncertainties to provide preferences over the objects modelled by expert systems. As a natural extension, intuitionistic fuzzy logic express mathematical support to model uncertainty of events in many practical situations. Over the large group of operators provided by Atanassov's Intuitinistic Fuzzy Logic (A-IFL) [1], necessity and possibility operators are considered in order to study their proprieties which are also valid to the classes of triangular (co)norms (t-conorms).

2 Intuitionistic Fuzzy Logic

An intuitionistic fuzzy set (A-IFS) A_I in a non-empty universe $\mathcal{X}$, expressed as

$$A_I = \{(\mathrm{x}, (\mu_{A_I}(\mathrm{x}), \nu_{A_I}(\mathrm{x}))) : \mathrm{x} \in \mathcal{X}, \mu_{A_I}(\mathrm{x}) + \nu_{A_I}(\mathrm{x})) \leq 1\},$$

extending a fuzzy set $A_I = \{(x, \mu_{A_I}(x), 1 - \mu_{A_I}(x)) : \mathrm{x} \in \mathcal{X}\}$, since the non-membership degree (nMD) $\nu_{A_I}(x)$ of an element $\mathrm{x} \in \mathcal{X}$ is less, at most equal to its complement, the membership degree (MD) $\mu_{A_I}(\mathrm{x})$. So, it does not necessarily equal to one [1].

Let $\tilde{U} \subset [0,1] \times [0,1]$, $\tilde{U} = \{\tilde{x} = (x_1, x_2) \in \tilde{U} : x_1 + x_2 \leq 1\}$ be the set of all pairs of MDs and nMDs with the order relation $\tilde{x} \leq_{\tilde{U}} \tilde{y}$ given by $x_1 \leq y_1$ and $x_2 \geq y_2$ such that $\tilde{0} = (0,1) \leq_{\tilde{U}} \tilde{x}$ and $\tilde{1} = (1,0) \geq_{\tilde{U}} \tilde{x}$, for all $\tilde{x}, \tilde{y} \in \tilde{U}$. The intuitionistic fuzzy connectives named as **negation, necessity and possibility** and defined by functions $N_I, \Box, \Diamond : \tilde{U} \to \tilde{U}$, respectively given by the expressions [3]:

$$N_I(\tilde{x}) = (x_2, x_1), \quad \Box(\tilde{x}) = (x_1, 1 - x_1) \quad \text{and } \Diamond(\tilde{x}) = (1 - x_2, x_2), \forall \tilde{x} = (x_1, x_2) \in \tilde{U}. \tag{1}$$

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LetN_I be an intitionistic fuzzy negation. By [3], for $\tilde{\mathbf{x}} = (\tilde{x}_1, \ldots, \tilde{x}_n) \in \tilde{U}^n$, the **$N_I$-dual intuitionistic function of $\tilde{f} : \tilde{U}^n \to \tilde{U}$**, denoted by $\tilde{f}_{N_I} : \tilde{U}^n \to \tilde{U}$, is given by:

$$\tilde{f}_{N_I}(\tilde{\mathbf{x}}) = N_I(\tilde{f}(N_I(\tilde{x}_1), \ldots, N_I(\tilde{x}_n))). \tag{2}$$

By [2], an **automorphism** $\phi : \tilde{U} \to \tilde{U}$ is a bijective, strictly increasing function: $\tilde{x} \leq \tilde{y}$ iff $\phi(\tilde{x}) \leq \phi(\tilde{y})$, $\forall \tilde{x}, \tilde{y} \in \tilde{U}$. For all $\tilde{\mathbf{x}} = (\tilde{x}_1, \ldots, \tilde{x}_n) \in \tilde{U}^n$, the **$\phi$-conjugate intuitionistic function of $\tilde{f}$**, denoted by $\tilde{f}^{\phi} : \tilde{U}^n \to \tilde{U}$, is given by:

$$\tilde{f}^{\phi}(\tilde{\mathbf{x}}) = \phi^{-1}(\tilde{f}(\phi(\tilde{x}_1), \ldots, \phi(\tilde{x}_n))). \tag{3}$$

Let $T \in \{T_P, T_M, T_L\}$ be an intuitionistic fuzzy t-norm and $S \in \{S_P, S_M, S_L\}$ be an intuitionistic t-conorm such that $T_P, T_M, T_L, S_P, S_M, S_L : \tilde{U}^2 \to \tilde{U}$ are defined as follows:

$$T_P(\tilde{x}, \tilde{y}) = (x_1y_1, x_2 + y_2 - x_2y_2) \quad S_P(\tilde{x}, \tilde{y}) = (x_1 + y_1 - x_1y_1, x_2y_2)$$
$$T_L(\tilde{x}, \tilde{y}) = (max(0, x_1 + y_1 - 1), min(1, x_2 + y_2))$$
$$S_L(\tilde{x}, \tilde{y})) = (min(1, x_1 + y_1), max(0, x_2 + y_2 - 1))$$
$$T_M((\tilde{x}, \tilde{y})) = (min(x_1, x_2), max(x_2, y_2)) \quad S_M(\tilde{x}, \tilde{y}) = (max(x_1, y_1), min(x_2, y_2))$$

Proposition 2.1. *If N_I is the function in Eq.(1a), for all $\tilde{x}, \tilde{y} \in \tilde{U}$, it holds that:*

$$(\Box \circ S)_{N_I}(\tilde{x}, \tilde{y}) = S_{N_I}(\Diamond(\tilde{x}), \Diamond(\tilde{y})) \qquad (\Box \circ T)_{N_I}(\tilde{x}, \tilde{y}) = T_{N_I}\Diamond(\tilde{x}, \Diamond(\tilde{y})) \tag{4}$$
$$(\Diamond \circ S)_{N_I}(\tilde{x}, \tilde{y}) = S_{N_I}(\Box(\tilde{x}), \Box(\tilde{y})) \qquad (\Diamond \circ T)_{N_I}(\tilde{x}, \tilde{y}) = T_{N_I}\Box(\tilde{x}, \Box(\tilde{y})) \tag{5}$$

Proposition 2.2. *If $\phi : \tilde{U} \to \tilde{U}$ is an automorphism, for all $\tilde{x}, \tilde{y} \in \tilde{U}$, the following holds:*

$$(\Box \circ S)^{\phi}(\tilde{x}, \tilde{y}) = S^{\phi}(\Box(\tilde{x}), \Box(\tilde{y})) \qquad (\Box \circ T)^{\phi}(\tilde{x}, \tilde{y}) = T^{\phi}\Box(\tilde{x}, \Box(\tilde{y})) \tag{6}$$
$$(\Diamond \circ S)^{\phi}(\tilde{x}, \tilde{y}) = S^{\phi}(\Diamond(\tilde{x}), \Diamond(\tilde{y})) \qquad (\Diamond \circ T)^{\phi}(\tilde{x}, \tilde{y}) = T^{\phi}\Diamond(\tilde{x}, \Diamond(\tilde{y})) \tag{7}$$

3 Conclusion

The operators given as the N_I-dual intuitionistic function and the ϕ-conjugate intuitionistic function of both intuitionistic fuzzy t-norm and t-conorm are preserved by the necessity and possibility connectives in the A-IFL. Other operators as robustness and correctness have been studied in further works [2].

References

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