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A WENO Wavelet Scheme for Solving Hyperbolic Equations of Two-phase Flows

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1 Introduction

Multiresolution or wavelet based methods have been successfully applied for solving hyperbolic systems of conservation laws [2–4]. One of the reasons for this success is the flexibility of the wavelet functions to be considered as sensors for detecting significant variations of the solutions. When applied to generate the sparse representation of the solutions at each time step, these wavelet sensors become the main ingredient for building adaptive schemes. In this work we consider the adaptive multiresolution method proposed in [1] to solve Riemann problems for systems of PDE's describing multiphase flows [5]. It is widely accepted that two-phase flow problems are non-hyperbolic and generally given in a non-conservative form. Nevertheless, some simplified models, as in [5], can be formulated in hyperbolic conservative form, with a complete set of real eigenvalues. This work deals with these models [5] whose mass, momentum and energy balances describe a 1D isentropic gas-liquid two-phase flow with a velocity difference between the two phases.

2 A WENO wavelet scheme

The numerical scheme applied in the current work was initially proposed in [1], where multi-species kinematic flow models were successfully solved. The main idea of the considered methodology is to apply a TVD Runge-Kutta scheme for the temporal integration associated to the Lax-Friedrich flux splitting approach for treating the spatial quantities. The numerical scheme (third order in time and fifth order in space) is applied to one dimensional two-phase problems. The numerical flux is obtained by the fifth order WENO

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scheme computed on a nonuniform grid. The grid adaptation is due to a thresholded interpolating wavelet transform that generates a sparse point representation of the solution at each time step. The nonuniform grid presents a smooth transition between scales because of the insertion of auxiliary values, helping to detect directions the information might travel within the domain [3]. Numerical results are presented and qualitatively compared with well established methodologies for two-phase flow problems. One of the main contributions of the present work is the increase of the solution sharpness while capturing shocks and rarefaction waves more accurately than the solutions obtained by other schemes. Other significant contribution is the reduction of the computational effort and consequently reducing computational time to accurately compute the solution, since the operators are exactly solved only in a very reduced number of grid points.

3   Conclusion

This study characterizes a successful effort to apply wavelet adaptive high resolution schemes for solving two-phase flow problems. One dimensional cases have been accurately computed and simulation results were qualitatively compared with solutions obtained by standard first order upwind schemes. Substantial computational gain is obtained despite the overhead from constructing the sparse grid. Despite high compression levels of the grid – implying the reduction of the computational cost for solving exactly the differential operator – the L_1-norm errors remain relatively small, below 1%. The good results obtained here motivate us to further investigate this methodology in 2D problems.

Acknowledgment

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