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A Parallel Multiresolution Scheme for Solving 2D Euler Equations of Gas Dynamics

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1 Introduction

Shock capturing numerical schemes is a wide research field which, among several formulations, includes high order finite difference and finite volume schemes. To achieve high order accuracy and convergence to the entropy solution, a set of possible choices are the essentially non-oscillatory (ENO) [3] and weighted essentially non-oscillatory (WENO) schemes [2], designed as an adaptive and as a nonlinear weight procedure to produce high order approximations in space, in order to preserve non-oscillatory shock transitions [1].

The current study presents a parallel algorithm based on the MPI protocol for the multiresolution scheme proposed in [4]. To validate the numerical scheme, a well established set of benchmark problems for this field is solved, namely the 2D Euler equations of gas dynamics. The present parallel version of the original serial scheme [4] now includes both ENO and WENO interpolating procedures to compute the space flux approximations. For the time discretization, the TVD Runge-Kutta scheme [4] is considered, and the flux splitting is done such that each space dimension can be solved independently for each row and/or column from the discrete domain. This provides a natural formulation for parallelization. In each space dimension, an 1D wavelet transform is applied to allow grid adaptation with respect to the solution at each time step.

2 Parallel Algorithm

The present parallel algorithm has been devised to exploit the fact that the splitting scheme for 2D problems at each time step can be solved independently for each row and

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column. The MPI protocol has been chosen for been relatively easy to program, resulting in a highly portable, scalable and device independent code. Workload balancing is achieved by using a master-slave strategy as follows. The code is executed on a MPI virtual machine composed by 1 master and N slave nodes. The master node stores all data and executes the main loop over time steps, distributing rows (columns) of data packages among the N slave nodes for specific data processing. The data packages are randomly distributed among slave nodes based on a first-free/first-served principle: a slave node, having received a data package, is considered occupied or non-free until the return of the processed data back to the master node has been accomplished. The simulations have shown that, independently of the grid sizes or the number of slaves, the execution performance is optimal when data messages are comprised by groups of 8 rows (columns). The proposed approach differs significantly from the parallel adaptive mesh refinement scheme given in [5].

3 Conclusions

Our parallel algorithm has been tested with 7 of the benchmark problems. The simulations were executed using either ENO or WENO schemes for 5 different initial grid sizes: $\eta = n_x = n_y = 2^8, \ldots, 2^{12}$. The reduction obtained by the sparse grid translates into execution time reduction and, when combined with the parallelism provided by the MPI approach (12 processors), leads to considerable speedups, ranging from $\sim$11 to $\sim$17. The relative cost of building a reduced sparse grid representation and reconstructing the grid averages to $\sim$15% while the MPI cost averages to $\sim$35% of the total execution time.

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