Unidimensional cutting stock problem: Mathematical models to minimize the number of saw cycles and objects

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1   Introduction

The minimization of the total waste or the total number objects cut is the most common criterion used when solving the Cutting Stock Problem (CSP)(*e.g.* [2]). Another important criterion is to maximize the cutting machine productivity which can be improved by reducing the number of different cutting patterns, *e.g.* [1] or taking into account that some cutting machines allow the objects to be stacked so that they can be cut simultaneously. The time necessary to adjust the cutting machine and to cut a stack of objects according to a given cutting pattern is named as *saw cycle* [4]. Mathematical model and heuristic solutions methods have been proposed in the literature so solve the CSP minimizing the total number of cycles and objects (CSP-CO) [3]. In this work we study four mathematical models for the CSP-CO considering the unidimensional case. To the best of our knowledge these models have not been compared before. For further work it is important to define if the proposed models are appropriated to represent the problem considering the solution quality and the associated solution time.

2   Mathematical Models for the CSP-CO and a Computational Study

The cutting stock problem (CSP-CO) considered here can be stated as: define how to cut a set of objects (all of size L and of thickness t) to produce a set of m items of size $l_i \leq L$ to attend a demand b_i for each item i, $i = 1...m$, and minimizing the total number of objects cut and the total number of saw cycles. The objects are available in stock in enough quantity. Given the saw height, h, the maximum number of objects that can be stacked in the machine is: $p = \left\lfloor \frac{h}{t} \right\rfloor$. All of the mathematical models considered here are

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extensions of the classical model proposed by Gilmory and Gomory in 1961 (*e.g* [3]). To present them, suppose that there are n possible cutting patterns that have been generated *a priori*. The patterns can be represented by A_j, a m-dimensional column vector with each element a_{ij} being the number of items i in pattern j, $j = 1, ..., n$. Let x_j and y_j be the number of objects and cycles associated to the cutting pattern A_j respectively, and γ_{sj} be the number of cycles with (s) objects being cut simultaneously, $s = 1 \dots p$. The Model M1 [4] is given by (1) subject to (2)-(3), and (8); the Model M2 is obtained including constraints (4) in Model M1; Model M3, adapted from [3] is obtained including constraints (5) in Model M1. The Model M4 is given by (6) subject to (7) and (8). Constraints (2), (4) and (7) guarantee that the demand is satisfied with overproduction and constraint (5) imposes that, if a cutting pattern j is used ($\lambda_j = 1$) then its frequency should be above f_{min}. The parameter M is a massive number.

$$min \qquad z = \quad \sum_{j=1}^{n} x_j + \sum_{j=1}^{n} y_j. \qquad\qquad\qquad (1)$$

$$\sum_{j=1}^{n} a_{ij}x_j \geq b_i, \qquad\qquad i = 1, ..., m. \qquad (2)$$

$$x_j \leq py_j, \qquad\qquad j = 1, \dots, n. \qquad (3)$$

$$\sum_{j=1}^{n} a_{ij}y_j \geq \lceil b_i/p \rceil, \qquad\qquad i = 1...m \qquad (4)$$

$$f\_min * \lambda_j \leq x_j \leq M * \lambda_j, \qquad\qquad j = 1, \dots, n. \qquad (5)$$

$$min \quad z = \quad \sum_{j=1}^{n} \sum_{s=1}^{p} s\gamma_{sj} + \sum_{j=1}^{n} \sum_{s=1}^{p} \gamma_{sj}. \qquad (6)$$

$$\sum_{j=1}^{n} \sum_{s=1}^{p} sA_j\gamma_{sj} \geq b_i, \qquad\qquad i = 1, \dots, m. \qquad (7)$$

$$x_j, y_j, \gamma_{sj} \geq 0 \text{ and integer}, \lambda_j = 0/1 \qquad s = 1, \dots, p; j = 1, \dots, n. \qquad (8)$$

A computational study was conducted with 90 instances from the literature. The results indicate that the model M2, that includes redundant constraints, is quicker than M1 for 62% of instances, and slower than M3 and M4 for 29% and 13% of the instances.

Referências

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