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# A Linear Integer Mathematical Model for Stowage Planning with Ship Stability

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**Abstract**. The contribution of this paper is the formulation of a new mathematical formulation which brings a linear bi- objective function to reduce total number of container movements and also improve container ship stability issues. Since each movement costs at most, depending on port, US$200, then minimization of number of movements is related with economic aspects. In addition, concerns about typical ship stability measures, like metacentric height and angle list, has been considered in the proposed approach and help to prevent ship capsize. The results attests the validity of proposed model and shows the impact of each objective function in container ship arrangement through ports.

**keywords**. Integer Programming, Stowage Planning, Ship Stability

## 1 Introduction

The container ship stowage planning problem consists in defining how the containers will be organized inside a ship along ports. For such purpose, two container features impacts on the number of movements necessary to unload and load a ship: container position on ship and its port destination. Since the container ship has a regular structure, containers are organized in stacks as described in Figure 1.

A container in the bottom of a stack could be unloaded only by removing all ones on top of it. Although, removed containers which destination are forward ports should be reloaded to container ship resulting on additional number of movements. In some cases, it is possible to avoid this additional number of movements employing a better stowage planning. Some articles in literature provide some discussion, mathematical models and heuristics to minimize total number of movements $[1, 2, 5]$.

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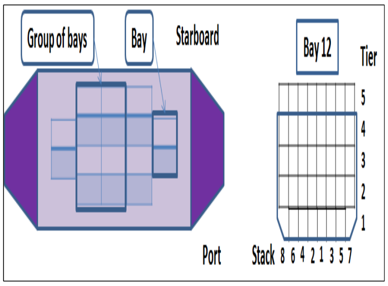


Figure 1: Container ship structure and spaces to store containers.

Another important issue is ship stability which can make the container ship to capsize and increase resistance through water [4]. These two problems could be avoided by changing containers arrangement to increase the metacentric height ($GM$) and reduction of angle list value ($\theta$), as illustrated in Figure 2.

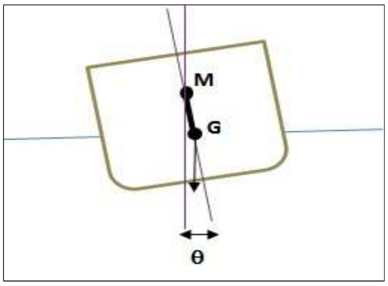


Figure 2: Metacentric ($M$), Geometric centre ($G$), and angle list ($\theta$) for a container ship.

Some references addressed stability problems, but without presenting a proper mathematical model integrated with minimization of number of movements [6], or using an non-linear terms to consider stability [2, 3].

This paper avoids the problem of using non-linear objective function for ship stability and proposes a model which tackles ship stability through a linear objective function. Thus, the contribution of this paper is to the first time present a bi-objective linear integer model to minimize the total number of movements and maximize stability measures. Additionally, some results about impact on container ship arrangement when stability issues are incorporated validates the mathematical model are presented.

## 2 Mathematical Model

The following assumptions have been made for the sake of simplicity, without compromising the solution general application:

 

1. The container ship has a rectangular format and is represented by a matrix with rows ($r = 1, 2, \cdots, R$), columns ($c = 1, 2, \cdots, C$) and bays ($d = 1, 2, \cdots, D$) with maximum capacity of $R \times C \times D$ containers. An irregular format may be achieved by simply adding constraints which represent imaginary containers that occupy the same spaces during the whole voyage [5].
2. All containers have the same size and weight.
3. The ship starts to be loaded in port 1, where it arrives empty;
4. The ship visits ports $2, 3, \cdots, N$ such that the container ship will be empty in the last port, because the ship performs a circular route where port $N$, in fact, represents port 1.
5. In each port $i = 1, 2, \cdots, N$, the container ship can be loaded with containers whose destination are ports $i + 1, \cdots, N$.
6. The container ship can always carry all the containers available in each port and this will never exceed its capacity.

The mathematical model in terms of linear programming with binary variables for the 3D SP is given by (1)-(8).

$$\min f\left(x_{ijv}(r,c,d), y_i(r,c,d)\right) = \alpha\phi_1\left(x_{ijv}(r,c,d)\right) + \beta\phi_2\left(y_i(r,c,d)\right) \tag{1}$$

subject to

$$\sum_{v=i+1}^{j}\sum_{r=1}^{R}\sum_{c=1}^{C}\sum_{d=1}^{D} x_{ijv}(r,c,d) - \sum_{k=1}^{i-1}\sum_{r=1}^{R}\sum_{c=1}^{C}\sum_{d=1}^{D} x_{kij}(r,c,d) = T_{ij}$$

$$i = 1 : N-1; j = i+1 : N \tag{2}$$

$$\sum_{k=1}^{i}\sum_{j=i+1}^{N}\sum_{v=i+1}^{j} x_{kjv}(r,c,d) = y_i(r,c,d)$$

$$i = 1 : N-1; r = 1 : R; c = 1 : C; d = 1 : D \tag{3}$$

$$y_i(r,c,d) - y_i(r+1,c,d) \geq 0$$

$$i = 1 : N-1; r = 1 : R; c = 1 : C; d = 1 : D \tag{4}$$

$$\sum_{i=1}^{j-1}\sum_{p=j}^{N} x_{ipj}(r,c,d) + \sum_{i=1}^{j-1}\sum_{p=j+1}^{N}\sum_{v=j+1}^{p} x_{ipv}(r+1,c,d) \leq 1$$

$$j = 2 : N; r = 1 : R-1; c = 1 : C; d = 1 : D \tag{5}$$

$$x_{ijv}(r,c,d) = 0 \;\; or \;\; 1$$

$$i, j, v = 1 : N; r = 1 : R; c = 1 : C; d = 1 : D. \tag{6}$$

where the binary variable $x_{ijv}(r,c,d)$ is defined as follows: if, in port $i$, the compartment $(r,c,d)$ has a container whose destination is port $j$ and this container was moved

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in port $v$, then the variable assumes value 1; otherwise value 0 is assumed. The term compartment $(r, c, d)$ represents row $r$, column $c$ for the container ship bay $d$. Similarly, variable $y_i(r, c, d)$ is defined as follows: if, in port $i$, the compartment $(r, c, d)$ has a container; then the variable assumes value 1; otherwise value 0 is assumed. The objective function (1) is composed of two terms: the first is the total cost of moving a container and, the second is the sum of instability measures for the container ship configuration in each port. It is assumed that, for all ports, the container movement costs the same and is equal to one. Constraints (2) express the total number of containers that will shipped from port $i$ to port $j$. Constraints (3) require that each compartment $(r, c, d)$ of the container ship is always occupied by at most one container. Constraints (4) are related to the physical storage of the containers in the ship, and it imposes that, for each container in row $r + 1$, there be another container in the row $r$ for all $r = 1, \cdots, R - 1$. Constraints (5) define how a container can be unloaded from the ship in port $j$ by requiring that, if a container occupies the position $(r, c, d)$ at port $j$, and it will be unloaded, then, there are no containers above or the containers above have already been unloaded at previous ports. Finally, Constraints (6) could be modified to be constant and equal to $-1$ for some position $(r, c, d)$ at any port $i$ in order to represent a container ship with irregular bays. A more detailed discussion about mathematical model assumptions could be find in [2].

The two terms which compose the objective function (Eq. (1)) define two optimization criteria: the first term is a function of the number of containers moved, $\phi_1(x)$, and the second depends on how the container ship is organized in each port, $\phi_2(y)$. The two criteria are combined by values given by weights $\alpha$ and $\beta$ in a manner that forms a bi-objective optimization framework.

The term $\phi_1(x_{ijv}(r, c, d))$ assumes that for all ports, the container movement cost is the same and is equal to one which may be translated as Eq. (7).

$$\phi_1(x_{ijv}(r, c, d)) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \sum_{v=i+1}^{j-1} \sum_{r=1}^{R} \sum_{c=1}^{C} \sum_{d=1}^{D} x_{ijv}(r, c, d) \tag{7}$$

The term $\phi_2(y_i(r, c, d))$ refers to the container ship´s transverse stability and assumes that every container has the same mass and is equal to one. This term is to control the container ship transverse stability before leave the port which means after all loading movements had been performed as described by Eq. (8).

$$\phi_2(y_i(r, c, d)) = \sum_{i=1}^{N} (-\Delta GM_i + \Delta L_i) \tag{8}$$

 

Table 1: Transportation information for a five port example.

| | P2 | P3 | P4 | P5 |
|---|---|---|---|---|
| P1 | 2 | 5 | 0 | 0 |
| P2 | 0 | 2 | 3 | 1 |
| P3 | 0 | 0 | 2 | 2 |
| P4 | 0 | 0 | 0 | 1 |

where:

$$\Delta GM_i = \left( \sum_{r=1}^{R} \left( \sum_{d=1}^{D} \sum_{c=1}^{C} y_i(r,c,d) \right) \cdot (GY_{ship} - r + 0.5) \right),$$

$$\Delta L_i = hp_i + hn_i,$$

$$\left( \sum_{c=1}^{C} \left( \sum_{d=1}^{D} \sum_{r=1}^{R} y_i(r,c,d) \cdot (GX_{ship} - c + 0.5) \right) \right) = hp_i - hn_i,$$

$$hp_i, hn_i \geq 0$$

where the values $GY_{ship}$ and $GX_{ship}$ represent vertical and horizontal coordinates of gravity center of the ship, respectively. The variables $\Delta GM_i$ represent the variance in metacentric height $GM$ in each port $i$ after loading all containers. Since $GM$ is the distance between the centre of gravity of a ship and its metacentre, as much metacentric height is increased with a $\Delta GMi > 0$, it turns more difficult the ship to overturn. The variables $\Delta Li$ also helps with the reduction of angle of list after loading all containers. Angle list measures the vessel leaning to either port or starboard. More discussion about metacentric height increasing and reduction of angle of list and corresponding objective functions could be seen at [4, 6].

In addition to conditions (a)-(f), the number of containers that must be loaded at a certain port is given by a transportation matrix $T$ of dimension $(N-1) \times (N-1)$, whose element $T_{ij}$ represents the number of containers from port $i$ that must be transported to the destination port $j$. This matrix is an upper triangular matrix, since $T_{ij} = 0$ for every $i \geq j$.

## 3 Results

The mathematical model has been applied in a small example just to illustrate how container ship arrangement could be affected by stability measures and the proposed model had been successful to provide more stable ship arrangements through ports.

The numerical example consists on a container ship with dimensions $R = 4$, $C = 4$ and $D = 1$. The number of ports is $N = 5$, and in each port there are $K = 2$ quay cranes available for unloading and loading operations. It was also considered, for simplicity, each column as bay. Each element $T_{i,j}$ from transportation matrix gives the number of containers that should be loaded in port $i$ which destination is port $j$. The transportation matrix used in this example is shown in Table 1.

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Table 2: Number of unloading and loading movements per port.

| Port | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| loading | 7 | 6 | 4 | 1 | 0 | 18 |
| unloading | 0 | 2 | 7 | 5 | 4 | 18 |

The mathematical model had been solved using GUSEK (the GLPK Integer Optimizer v4.55 with all cuts enabled) and the corresponding model has 176 constraints, and 400 binary variables. The integer solution had been found in 0.4 seconds. When the model is set to minimize only the total number of container movements ($\alpha = 1$ and $\beta = 0$), the resulting container ship arrangement is given in Figure 3. A different container ship arrangement is produced for only stability measures maximization ($\alpha = 0$ and $\beta = 1$) as shown in Figure 4. Each square in both figures has a pair of numbers $(i, j)$ representing a space of the ship occupied by a container, or 0 representing an empty space. The first number of the pair gives the container loading port information and the second gives the container unloading port information. The number of movements per port for both solutions is presented in Table 2 which leads to a total cost of US$7200. The article [2] provides a detailed procedure on how to compute stability measure.

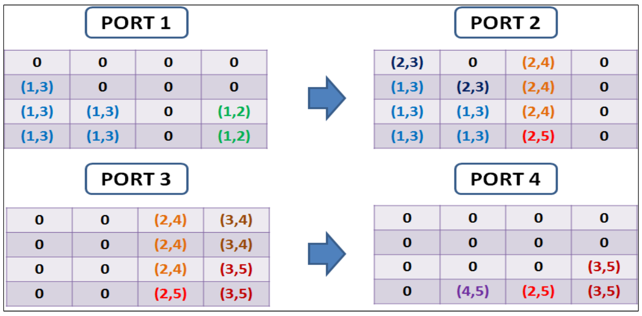


Figure 3: The container ship arrangement for minimization of $\phi_1(\cdot)$.

Figure 4 shows a more stable container ship arrangement specially in ports 2 and 3 when compared with the arrangement in Figure 3, but without an additional number of container movements.

## 4 Conclusions

This paper presented a new mathematical model that minimizes total number of container movements in a ship and also is able to address stability issues using just a linear objective function. A small numerical test had been performed and showed that the model is promising since it could give more stable container ship arrangements without additional container movements. Future works consist in test non-regular ship arrangement and ships with a larger capacity, like real ones could reach, with 18,000 containers.

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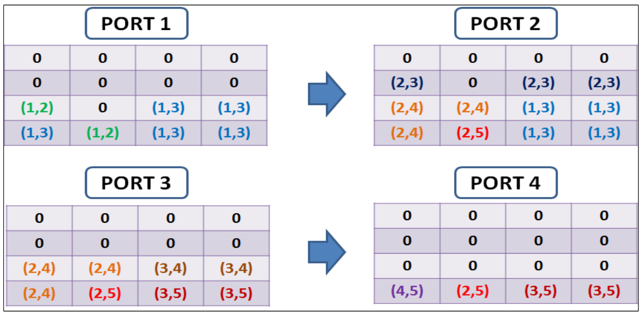


Figure 4: The container ship arrangement for minimization of $\phi_2(\cdot)$.

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