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Topographical Global Optimization Applied to an Inverse Radiative Transfer Problem: Preliminary Results

Lucas C. Silva Jardim[1]
Diego C. Knupp[2]
Antônio J. Silva Neto[3]
Departamento de Engenharia Mecânica e Energia, IPRJ/UERJ, Nova Friburgo, RJ
Wagner F. Sacco[4]
Instituto de Engenharia e Geociências, Universidade Federal do Oeste do Pará - UFOPA

The mathematical formulation of the radiative transfer problem is given by a integro-differential equation [3]. If the geometry, the boundary conditions and the radiative properties are known, the problem can be solved directly, yielding the radiation intensity for the whole spatial and angular domain. This is the direct problem. Supposing that either the geometry, the boundary conditions, the radiative properties, or a combination of them, are unknown, but experimental data on the radiation that leaves the medium are available, one may then try to estimate the unknowns using the available experimental data. This is the so called inverse radiative transfer problem.

The radiative transfer problem tackled in this work consists of a one-dimensional participating medium with space variable single scattering albedo, $\omega(\tau)$, modeled as a second order polynomial expansion:

$$\omega(\tau) = D_0 + D_1\tau + D_2\tau^2 \tag{1}$$

In order to solve the direct problem we have used Chandrasekhar's discrete ordinates method [1] in which the polar angle domain is discretized, and the integral term on the governing equation is replaced by a gaussian quadrature. The inverse problem is formulated implicitly [2] and it consists of a objective function $Q(\vec{Z})$ given by the sum of the squared residues between the estimated - provided by solving the direct problem - and the experimentally measured radiation. The vector $\vec{Z}$ contains the unknowns to be estimated. In this work $\vec{Z} = \{D_0, D_1, D_2, \tau_0\}$, where D_i, $i = 1, 2, 3$, are coefficients of the expansion and τ_0 is the optical thickness.

Hence, the inverse problem solution can be obtained by the minimization of this objective function. For this purpose, in this work, we propose the use of a clustering optimization method called Topographical Global Optimization (TGO), introduced by Törn and Viitanen [4], which can be briefly described by three steps:

[1]lucascjardim@gmail.com
[2]diegoknupp@iprj.uerj.br
[3]ajsneto@iprj.uerj.br
[4]wagner.sacco@ufopa.edu.br

1. Sample the search space with N uniformly distributed random points.
2. Construct of the topography by analyzing the objective function value at each one of the N points with respect to their k-nearest neighbors. When a point P_i has neighbors which evaluate to higher objective function values, this point P_i is considered to be the topography minimum.
3. All the topographical minima from Step 2 are set as initial solutions for a local optimization method. The global minimum is the lowest function evaluation from all the local search executions.

In the absence of real experimental data, simulated measurements were constructed with the direct problem solution and noise addition from a normal distribution. The test case presented in this work consists of $\vec{Z} = \{D_0 = 1, D_1 = 0, D_2 = -0.6, \tau_0 = 1\}$. Table 1 shows the results for five executions obtained with $N = 200$ initial points, $k = 10$ neighbors analyzed and the Nelder-Mead algorithm for the third Step, where NFE refers to the number of objective function evaluations. The results show that in most runs good solutions were obtained, with the estimates very close to the expected values, with a remarkable low number of objective function evaluations.

Table 1: Results obtained with $N = 200$ and $k = 10$.

Execution	$D_0 = 1,0$	$D_1 = 0,0$	$D_2 = -0,6$	$\tau_0 = 1$	$Q(\vec{Z})$	NFE
1	1,000	-0,013	-0,580	1,002	0,000004	323
2	0,914	0,490	-0,943	1,067	0,006403	330
3	1,000	-0,001	-0,599	1,00	0,000000	348
4	1,003	-0,004	-0,604	0,998	0,000019	305
5	0,961	0,311	-0,896	1,017	0,001361	320

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