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# Mathematical modeling in the airline industry: optimizing aircraft assignment for on-demand air transport

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**Abstract**. This paper addresses a problem faced by air companies that offer on-demand flight services. Given a list of flight requests, the company has to assign its aircrafts to these requests, so that operational costs are minimized. The main issue in this planning process regards the repositioning of aircrafts when they are not available at the airports of customer departure. The cost of this repositioning should be the least possible, as customers pay proportionally to their requested flights only. We propose an optimization model to support decision making in these situations. Computational experiments with real-life data provided by an on-demand air transport company indicate that the proposed model is appropriate and may significantly reduce the time spent on repositioning flights.

**Keywords**. airline industry, aircraft assignment, mathematical model, optimization.

## 1 Introdução

The airline industry is well known by its complexity and costly activities. To run efficiently, an air company must plan its operations very carefully, from aircraft choices to ticket prices. This need for efficiency has become even stronger in the last years, due to an increasing in competitiveness and to the current economic crisis. As a result, air companies are recurring to scientific methods to support decision making. Mathematical models and computational methods have became a must in their planning process [1, 2].

Another phenomenon observed in the last years concerns the increasing of on-demand air transport services, mainly represented by air taxi and fractional ownership companies [2–4]. Differently from the traditional commercial airlines these services are oriented to individual customers and do not work with pre-scheduled flights. Customers impose their departure times and airports and the company has to assign aircraft and crew to each customer request or to a group of them. The flights are point-to-point, outside hub airports and allow around six passengers. In *per-seat* services, the customer buys a single seat on the flight, while in *per-aircraft* services the customer gets the whole aircraft.

In this paper, we address the aircraft allocation problem that arises in the planning process of per-aircraft on-demand air transport services. This research have been motivated by a case study developed with an air company that offers this service to European

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countries. We propose an optimization model that supports decision making on aircraft allocation, in which the objective is to minimize the total time on repositioning flights.

The remainder of this paper is organized as follows. Section 2 describes the characteristics of the aircraft allocation problem. Section 3 gives the full description of a new optimization model that formulates this problem. Results of computational experiments using real-life data are presented and analyzed in Section 4. Finally, we present the conclusion and topics for future research on Section 5.

## 2 Problem description

Consider an air company that has a fleet of aircrafts of different types distributed over a number of airports. A customer contacts the company and requests a flight by choosing: (*i*) the aircraft type; (*ii*) origin and destination airports; (*iii*) departure time of the flight. The origin and destination airports determine a *live* leg. An aircraft is then assigned to this request to service the requested live leg. A *ferry* leg (a.k.a. *deadhead* or *non-revenue* flight) is and additional flight needed when no aircraft of the type requested is available at the departure airport. This repositioning flight incurs a high cost to the company, as the customer pays proportionally to flight hours concerning the live leg only. Typically, repositioning comprises 35% or more of the total flying time [4]. Therefore, the aircraft assignment should minimize the time spent on ferry legs.

The input data available at the planning process is the following: (1) A list of available airports; (2) A list of available aircrafts, such that each aircraft has a type, travel times between any two airports, waiting time required between flights, and the airport in which the aircraft is currently available; and (3) A list of a flight requests with their respective required departure times, aircraft types and origin and destination airports. The goal is to determine the assignment of aircrafts to customer requests that minimizes the total cost with ferry legs.

## 3 Mathematical model

In this section, we propose an optimization model that formulates the on-demand air transport problem described in the previous section. Different formulations have been proposed in the literature to deal with this problem. They are classified into two types: standard mixed-integer programming models and set partitioning models [3, 5]. While set partitioning models are recognized as the most suitable in practice, they require the implementation of advanced techniques, such as column generation and branch-and-price methods. On the other hand, standard mixed-integer programming models have the benefit of requiring no specific method, as the model can be solved straightforwardly by a general-purpose optimization software. However, the formulations proposed in the literature seem to be available for problems with few flight requests only, as high computational times are observed in problems with a reasonably large number of requests.

The formulation proposed in this paper is a standard mixed-integer programming model that is able to handle a reasonable large number of flights requests at once, without

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compromising the running time of the optimization software. As indicated by the results of computational experiments presented ahead in this paper, problems with more than 100 requests can be solved in a few minutes by a state-of-the-art software.

First, we define the notation and parameters required to describe the model. Consider the following sets: $\mathcal{P} = \{1, \ldots, P\}$ is the set of aircraft *types*; $\mathcal{V} = \{1, \ldots, N\}$ is the set of aircrafts – we can partition this set in subsets $\mathcal{V}_1, \ldots, \mathcal{V}_P$, according to the $P$ types of aircrafts; $\mathcal{R} = \{1, \ldots, R\}$ is the set of flight requests; $\mathcal{K} = \{1, \ldots, K\}$ is the set of airports. Using these sets, we define the following input parameters:

- $T^p_{ij} \in \mathbb{R}_+$: travel time between airports $i$ and $j$ for an aircraft of type $p$, for all $i, j \in \mathcal{K}$, $p \in \mathcal{P}$;
- $k_v \in \mathcal{V}$: initial airport of aircraft $v$, for all $v \in \mathcal{V}$;
- $TAT^r_k \in \mathbb{R}_+$: turn around time on airport $k$ before servicing request $r$. This is the waiting time required between flights, usually due to the boarding of passengers and aircraft setup.
- $ST_r \in \mathbb{R}_+$: starting time of request $r$, for all $r \in \mathcal{R}$;
- $\Delta \in \mathbb{R}_+$: maximum delay allowed for start servicing a request;
- $i_r, j_r \in \mathcal{V}$: origin and destination airports of request $r$, for all $r \in \mathcal{R}$;
- $p_r \in \mathcal{P}$: aircraft type required in request $r$, for all $r \in \mathcal{R}$.

Additionally, we state the following decision variables:

- $x_{vijr} = \begin{cases} 1, & \text{if aircraft } v \in \mathcal{V} \text{ goes directly from airport } i \text{ to } j \text{ for servicing} \\ & \text{request } r \in \mathcal{R}, \\ 0, & \text{otherwise.} \end{cases}$
- $y_{vrs} = \begin{cases} 1, & \text{if aircraft } v \in \mathcal{V} \text{ services requests } r, s \in \mathcal{R} \text{ consecutively,} \\ 0, & \text{otherwise.} \end{cases}$
- $a_{ir}$ = time instant in which an aircraft becomes available at airport $i \in \mathcal{K}$ for servicing request $r \in \mathcal{R}$.
- $d_{ir}$ = time instant in which an aircraft departures from airport $i \in \mathcal{K}$ for servicing request $r \in \mathcal{R}$.

Based on the notation and decision variables just defined, the proposed mixed-integer programming model for the on-demand air transport problem is the following:

$$\min \quad \sum_{r \in \mathcal{R}} \sum_{v \in \mathcal{V}} \sum_{\substack{k \in \mathcal{K} \\ k \neq i_r}} T^{p_r}_{k i_r} x_{v k i_r r} \tag{1}$$

$$\text{s.t.} \quad \sum_{v \in \mathcal{V}_{p_r}} x_{v i_r j_r r} = 1, \qquad \forall r \in \mathcal{R}, r > 0, \tag{2}$$

$$\sum_{\substack{k\in\mathcal{K}\\k>0}} x_{vki_rr} = x_{vi_rj_rr}, \qquad \forall v\in\mathcal{V}, \forall r\in\mathcal{R}, r>0, \tag{3}$$

$$x_{v0k_v0} = 1, \qquad \forall v\in\mathcal{V}, \tag{4}$$

$$x_{v0j0} = 0, \qquad \forall v\in\mathcal{V}, \forall j\in\mathcal{K}, j\neq k_v, \tag{5}$$

$$x_{vi_rjr} = 0, \qquad \forall v\in\mathcal{V}, \forall r\in\mathcal{R}, r>0, \forall j\in\mathcal{K}, j\neq j_r, j\neq i_r, \tag{6}$$

$$x_{vij_rr} = 0, \qquad \forall v\in\mathcal{V}, \forall r\in\mathcal{R}, r>0, \forall i\in\mathcal{K}, i\neq i_r, \tag{7}$$

$$x_{vijr} = 0, \qquad \forall v\in\mathcal{V}, \forall r\in\mathcal{R}, r>0, \forall i,j\in\mathcal{K}, i\neq i_r, i\neq j_r, j\neq i_r, j\neq j_r, \tag{8}$$

$$x_{vijr} = 0, \qquad \forall r\in\mathcal{R}, r>0, \forall v\notin\mathcal{V}_{p_r}, \forall i,j\in\mathcal{K}, \tag{9}$$

$$\sum_{\substack{k\in\mathcal{K}\\k>0}} x_{vk00} = 1, \qquad \forall v\in\mathcal{V}, \tag{10}$$

$$x_{vi_rj_rr} = \sum_{\substack{s\in\mathcal{R}\\s\neq r}} y_{vrs}, \qquad \forall v\in\mathcal{V}, \forall r\in\mathcal{R}, r>0, \tag{11}$$

$$\sum_{\substack{r\in\mathcal{R}\\r\neq s}} y_{vrs} = \sum_{k\in\mathcal{K}} x_{vki_ss}, \qquad \forall v\in\mathcal{V}, \forall s\in\mathcal{R}, s>0, \tag{12}$$

$$y_{vrs} \leq x_{vj_ri_ss}, \qquad \forall v\in\mathcal{V}, \forall r,s\in\mathcal{R}, r>0, s>0, r\neq s, \tag{13}$$

$$\sum_{\substack{s\in\mathcal{R}\\r\neq s}} \sum_{v\in\mathcal{V}} y_{vrs} = 1, \qquad \forall r\in\mathcal{R}, r>0, \tag{14}$$

$$\sum_{s\in\mathcal{R}} y_{v0s} = 1, \qquad \forall v\in\mathcal{V}, \tag{15}$$

$$\sum_{r\in\mathcal{R}} y_{vr0} = 1, \qquad \forall v\in\mathcal{V}, \tag{16}$$

$$y_{v0s} \leq x_{vk_vi_ss}, \qquad \forall v\in\mathcal{V}, \forall s\in\mathcal{R}, s>0, \tag{17}$$

$$y_{vr0} \leq x_{vj_r00}, \qquad \forall v\in\mathcal{V}, \forall r\in\mathcal{R}, r>0, \tag{18}$$

$$y_{v00} \leq x_{vk_v00}, \qquad \forall v\in\mathcal{V}, \tag{19}$$

$$ST_r \leq d_{i_rr} \leq ST_r + \Delta, \qquad \forall r\in\mathcal{R}, r>0, \tag{20}$$

$$d_{kr} \geq a_{kr}, \qquad \forall k\in\mathcal{K}, \forall r\in\mathcal{R}, r>0, k>0, \tag{21}$$

$$a_{i_rr} \geq d_{kr} + (T^{p_r}_{ki_r} + TAT^r_{ir}) x_{vki_rr}, \qquad \forall r\in\mathcal{R}, r>0, \forall v\in\mathcal{V}, \forall k\in\mathcal{K}, k\neq i_r, k\neq j_r, \tag{22}$$

$$a_{ks} \geq d_{i_rr} + T^{p_r}_{i_rj_r} + TAT^s_k + {} + M_{krs}\left(\sum_{v\in\mathcal{V}} y_{vrs} - 1\right), \qquad \forall k\in\mathcal{K}, \forall r,s\in\mathcal{R}, k>0, r\neq s, s>0, \tag{23}$$

$$x_{vijr}, y_{vrs} \in \{0,1\}, \qquad \forall v\in\mathcal{V}, \forall i,j\in\mathcal{K}, \forall r\in\mathcal{R}, \tag{24}$$

$$a_{ir}, d_{ir} \geq 0, \qquad \forall i\in\mathcal{K}, \forall r\in\mathcal{R}, \tag{25}$$

 

where $M_{krs} = ST_r + T^{p_r}_{i_r j_r} + TAT^s_k$ is a Big-$M$ constant. The objective function (1) is given by the total travel time spent on ferry legs (aircraft repositioning), which corresponds to the extra cost for the company. Constraints (2) ensure that all request $r \in \mathcal{R}$ are serviced by exactly one aircraft of type $p_r$ as required by the customer. Constraints (3) impose that to service request $r$, the aircraft must come from an airport (which can be the same as the current one in case $k = i_r$). We assume that all aircrafts are initially available at a dummy airport 0 so that each aircraft $v$ must go from 0 to its actual initial airport $k_v$ and then return to the dummy 0 at the end of its route. This is imposed by constraints (4)–(10). Constraints (11) and (12) link variables $x$ and $y$ using the fact that if a given aircraft $v$ services request $r$ by visiting airports $i_r$ and $j_r$, then it must service another request $s$ in the sequence. We include a dummy request 0 in set $\mathcal{R}$, so that the first and last requests in the aircraft route also have predecessor and successor requests. We then relate the two types of variables by using a flow of the aircraft from $x$ to the corresponding $y$ (and vice-versa). Constraints (13) impose that if aircraft $v$ services requests $r$ and $s$ consecutively (*i.e.*, $y_{vrs} = 1$), then it must go from the last airport of $r$ (*i.e.*, $j_r$) to the initial airport of $s$ (*i.e.*, $i_s$). Constraints (14) guarantee that each request $r$ is serviced by exactly one aircraft, while constraints (15) and (16) impose that the route of any aircraft must start and finish at the dummy request 0, respectively. If $s$ is the first request serviced by aircraft $v$, then the first airport visited on this request must be the initial airport of $v$ (*i.e.*, $k_v$), as imposed by constraints (17). If $r$ is the last request serviced by aircraft $v$, then the aircraft must go from the last airport of this request (*i.e.*, $j_r$) to the dummy airport 0, which is stated in constraints (18). Also, if an aircraft is not used, (19) imposes that it must go from the dummy request to itself and from its initial airport to the dummy airport, as a convention. Constraints (20) ensure that the departure time of the aircraft that services request $r$ satisfies the schedule imposed by the customer. Notice that a delay of $\Delta$ minutes is allowed. Constraints (21) guarantee that the aircraft visiting the first airport of request $r$ can departure from this airport only after the time instant that this aircraft becomes available. This time instant is determined by constraints (22), for the cases in which the aircraft visits the first airport of request $r$ after repositioning (*i.e.*, the aircraft comes from an airport $k \neq ir$). These constraints relates different airports of a same request. Constraints (23) impose the time instant in which the aircraft becomes available using two different requests. When a request $s$ succeeds a request $r \neq s$, the destination airport of $r$ (*i.e.*, $j_r$), is the first airport that is used to service request $s$ (*i.e.*, either we have $i_s = j_r$ or there is a positioning from $k = j_r$ to $i_s$). Hence, the time instant from which the aircraft that visits the first airport of request $s$ becomes available is obtained using the departure time from $j_r$ plus the travel time and the turn around time. In case $s$ does not succeeds $r$, the constraint must be turned off, which is done by using the Big-$M$ constant and the summation of $y_{vrs}$ over $v$.

## 4 Computational experiments

In this section, we present the results of computational experiments using the proposed optimization model. The model was implemented on top of IBM CPLEX Optimization

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solver v. 12.4, using the IBM CPLEX Concert library for C++. All the input data is provided by means of text files, except for the travel times between airports, which are computed internally by using the Great Circle distance. All experiments were run on a Linux PC with a processor Intel Core i7-4790 3.6 GHz and 16 GB of memory.

A case study was developed on a company that offers on-demand air transport services in Europe. The company provided journey logs from their operation on 10 consecutive days of three different months of 2014. The live requests correspond to customer flight requests and pre-scheduled aircraft maintenance operations. From these data, we have generated 12 instances by creating one instance for each seven consecutive days, for each month. Hence, we have obtained four instances for each of the months. The first six columns of Table 1 describe the characteristics of the instances considered in the experiments. The headers follow the notation defined on Section 3.

Tabela 1: Problem instances and comparison between the company policy and the results obtained by the optimization model.

| | | | | | | Ferry legs (repositioning) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Company | Model, $TAT = 0$ | | Model, $TAT = 30$ | |
| Month | Days | $R$ | $K$ | $N$ | $P$ | Time (h) | Time (h) | Difference | Time (h) | Difference |
| 1 | 1 to 7 | 76 | 35 | 25 | 5 | 61.75 | 53.80 | 7.95 | 55.73 | 6.02 |
| | 2 to 8 | 76 | 36 | 26 | 5 | 52.08 | 45.40 | 6.68 | 47.33 | 4.75 |
| | 3 to 9 | 82 | 40 | 28 | 5 | 56.90 | 50.33 | 6.57 | 52.27 | 4.63 |
| | 4 to 10 | 84 | 44 | 27 | 5 | 62.42 | 57.12 | 5.30 | 59.05 | 3.37 |
| 2 | 11 to 17 | 89 | 48 | 28 | 6 | 82.25 | 64.27 | 17.98 | 65.45 | 16.80 |
| | 12 to 18 | 80 | 49 | 27 | 6 | 64.72 | 54.32 | 10.40 | 54.32 | 10.40 |
| | 13 to 19 | 81 | 51 | 25 | 6 | 61.42 | 51.65 | 9.77 | 51.65 | 9.77 |
| | 14 to 20 | 76 | 50 | 25 | 6 | 57.23 | 47.73 | 9.50 | 47.93 | 9.30 |
| 3 | 6 to 12 | 88 | 48 | 31 | 6 | 122.02 | 117.92 | 4.10 | 122.85 | -0.83 |
| | 7 to 13 | 93 | 50 | 31 | 6 | 136.17 | 132.02 | 4.15 | 136.95 | -0.78 |
| | 8 to 14 | 96 | 51 | 30 | 6 | 130.82 | 126.12 | 4.70 | 129.33 | 1.48 |
| | 9 to 15 | 102 | 52 | 30 | 6 | 137.92 | 137.92 | 0.00 | 142.05 | -4.13 |

Columns 7 to 11 of Table 1 show the total time in hours spent on ferry legs according to three cases: the actual aircraft allocation used by the company in practice (Company) and the allocation obtained from the optimal solution of the proposed optimization model with two different turn around times ($TAT = 0$ and $TAT = 30$). In the last two cases we present the respective time of ferry legs and its difference to the company time. All instances were solved to optimality in less than 10 minutes.

The results obtained by the model with $TAT = 30$ indicate the advantages of using an optimization approach to aircraft allocation. Time reductions of up to 16 hours were obtained for a planning horizon of 7 days. This implies in a significant cost reduction for the company, as flying costs go from \$3000 to \$8000 euros per hour, depending on the aircraft type. Notice that for some instances of month 3, the results provided by the model with $TAT = 30$ had ferry times worse than those executed by the company allocation. This happens because the TAT applied in practice may be very small in some situations. For example, in the last instance of month 3 (days 9 to 15) we observed some actual turn around times equal to 5 minutes. This sensitivity of TAT to certain situations is a topic of future research. To verify the impact of using different TAT values, we have included the results with $TAT = 0$ in Table 1.

 

## 5 Conclusions

We addressed a problem that arises in the planning process of on-demand air transport companies. Given a list of flight requests, the company has to assign aircrafts to service these requests. As aircrafts may not be readily available on the departure airports required by customers, the planning should include repositioning flights, which impose additional costs to the company, as customers pay proportionally to the requested flight hours. We proposed an optimization model to support the decision making process in such situations. The model determines optimal routes to aircrafts so that all customer requests are satisfied and the flight hours on ferry legs are minimized. Computational experiments using real-life data indicates the advantages of using the proposed optimization model, resulting in reductions of up to 16 hours in repositioning time.

It is worth mentioning that the proposed model considers a simplified version of the real situation faced by the company. As future research on a ongoing project, additional features will be included in the model, as an explicit dealing of maintenance requests, crew allocation and the possibility of upgrades on the customer requests. These features should result in a more realistic and flexible model.

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