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Simulation of phytoplankton and nutrients interaction in a shallow lake

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1 Introduction

The Ypacarai lake, in Paraguay, is one of the most important water resources. At the moment, its physicochemical and biological characteristics show that its aquatic ecosystems are degraded in eutrophic state. Furthermore, the large amount of effluents discharged in the lake is the main cause of its pollution, because they carry nutrients, in the form of nitrogen and phosphorus, which promote algae (phytoplankton) growth. In this work, we analyze the interaction between phytoplankton and nutrients (nitrogen and phosphorus) getting into the lake through the tributaries, under different wind conditions. This analysis includes the kinetics of algae growth to predict zones with high eutrophication.

2 Modeling

The hydrodynamics of the lake is modeled through the 2D shallow water equations, where equation (1) is the continuity equation and equation (2) is the momentum equation. We include the wind and bottom stresses only and neglect the Coriolis and turbulence terms. Equation (3) shows the depth averaged scalar transport equation, to model total phosphorus, total nitrogen and phytoplankton.

$$\frac{\partial h}{\partial t} + \frac{\partial (u_i h)}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -g \frac{\partial}{\partial x_i}(h + z_0) + \frac{1}{h\rho} \frac{\partial}{\partial x_j}(h\tau_{ij}) - \frac{1}{h} \frac{\tau_{so,i}}{\rho} + \frac{1}{h} \frac{\tau_{wind,i}}{\rho} \tag{2}$$

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$$\frac{\partial(hc_i)}{\partial t} + u_i \frac{\partial(hc_i)}{\partial x_j} = D \frac{\partial^2}{\partial x_j^2}(hc_i) + g_i + f_i \tag{3}$$

where $u_i \colon i \in \{1, 2\}$ are the depth-averaged flow velocities (u, v) in x and y, h is the water depth, z_0 the bed elevation, τ is the momentum diffusion term, τ_{so} is the horizontal component of the bed friction, τ_{wind} is the component of the wind stress, ρ is the water density and g the gravity acceleration. Finally, $c_i \colon i \in \{1, 2, 3\}$ are the scalar concentrations of nitrogen, phosphorus and chlorophyll a, D is the diffusion coefficient, g_i are the consumption rates of phosphorus and nitrogen, and production rate of chlorophyll a, and f_i includes other terms such as phosphorus sedimentation and inlets/outlets.

3 Conclusion

Simulated values of nutrients and algae distributions are contrasted with government measurements showing similar results. This analysis enables the detection of high polluted zones based on the complete distribution of algae in the lake. The results are useful in the definition of environmental management actions focusing on reducing damage to the aquatic and terrestrial ecosystem.

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