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# Wind effects on shallow water contaminant transport simulation

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## 1 Introduction

Ypacarai lake is an important water resource with great ecological value in Paraguay. It constitutes an important natural, tourist, cultural, recreational area and plays a key role in the development of industries and agriculture. Despite of its importance, human activity and the lack of environmental regulation and management have produced a severe increase on its pollution, to levels that produces a strong degradation of its surrounding biosphere [2, 3]. Several attempts to recover the lake have failed, mainly due to the lack of complete information about its dynamic and interaction with its boundaries (including its bottom) and meteorological components (such as the wind). In this work, we analyze the influence of wind and bottom in the distribution of a contaminant discharged into the lake to identify critical regions in it.

## 2 Modelling

The 2D shallow water equations (1) and (2) are used, focusing on the wind and bottom stresses only, and therefore, neglecting the Coriolis and turbulence terms. Equation (3) shows the depth averaged scalar transport equation that is solved coupled to the hydrodynamic equations.

$$\frac{\partial h}{\partial t} + \frac{\partial(u_i h)}{\partial x_i} = 0 \quad (1)$$

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$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -g \frac{\partial}{\partial x_i} (h + z_0) + \frac{1}{h\rho} \frac{\partial}{\partial x_j} (h\tau_{ij}) - \frac{1}{h} \frac{\tau_{so,i}}{\rho} + \frac{1}{h} \frac{\tau_{wind,i}}{\rho} \quad (2)$$

$$\frac{\partial(hc)}{\partial t} + u_i \frac{\partial(hc)}{\partial x_j} = D \frac{\partial^2}{\partial x_j^2} (hc) \quad (3)$$

where  $u_i$  ( $i = 1, 2$ ) are the depth-averaged flow velocities ( $u, v$ ) in  $x$  and  $y$ ,  $h$  is the water depth,  $z_0$  the bed elevation,  $\tau$  is the momentum diffusion term,  $\tau_{so}$  is the horizontal component of the bed friction,  $\tau_{wind}$  is the component of the wind stress,  $\rho$  is the water density and  $g$  is the gravity acceleration. Finally,  $c$  is the scalar concentration, and  $D$  is the diffusion coefficient. In a future work, we will consider chemical reactions.

The 3D geometry of the lake is obtained from satellite photography image processing considering bathymetric data, which is used to separate the geometry in different depth zones. The equations are discretized by finite volume method, and a new solver based on OpenFOAM is implemented to include the aforementioned effects.

### 3 Conclusion

Simulated hydrodynamic and concentration profiles are contrasted with government field data observing correlation between them. The results show areas in the lake that due to the configuration of the flow are of higher risk of contaminant accumulation.

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### References

- [1] J. Józsa. Shallow lake hydrodynamics. Theory, measurement and numerical model applications. *Lecture Notes of the IAHR short course on environmental fluid mechanics*, Budapest, pages 7-16, 2004.
- [2] L. Oporto, and C. E. Schaerer. Wind effects on contaminant transport in shallow lakes. *Proceeding Series of the XXXVI Iberian Latin-American Congress on Computational Methods in Engineering*, 2015.
- [3] L. Oporto, D. Ramirez, J. Varela, and C. E. Schaerer. Analysis of contaminant transport under wind conditions on the surface of a shallow lake. *Mecánica Computacional*, 34:2155-2163, 2016.
- [4] I. K. Tsanis, J. Wu, H. Shen, and C. Valeo. Environmental hydraulics: hydrodynamic and pollutant transport modelling of lakes and coastal waters. *Developments in Water Science*, 56:vii-viii,1-360, 2006.