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Comparison of some strategies for restarting GMRES

Gustavo Espínola[1]
Juan C. Cabral[2]
Pedro Torres López[3]
Christian E. Schaerer[4]
National University of Asuncion, San Lorenzo, Paraguay

1 Introduction

Restarted Generalized Minimal Residual Method (GMRES(m)) is one of the most successful methods for solving linear system of equations [7]. At each cycle, GMRES(m) uses the residual at the previous cycle as starting guess, and constructs a Krylov subspace of dimension m with $m \ll n$ (where n is the dimension of the linear system) for computing a new residual, which is used as the starting residual for the next cycle, i.e., the next call to a GMRES routine. Rate of GMRES(m) convergence depends on an appropriate election of the restarting parameter m. In this context several algorithms have been proposed for choosing statically and dynamically the parameter m or introducing vectors for enriching the subspace.

2 Models comparison

In this work, we compare three strategies (GMRES-E(m, d), LGMRES(m, l) and PD-GMRES(m)) for choosing iteratively an appropriate variation or enrichment of the Krylov subspace for improving GMRES(m) convergence. In [5] was proposed the GMRES-E(m, d) , where d approximate eigenvectors added to the Krylov subspace and in [6] was pointed out that augmenting with harmonic Ritz vectors leads to better convergence results. In [1] was introduced a new restarted augmented LGMRES(m, l) algorithm where the Krylov subspace augments with l previous approximations of the error, while in [4], the PD-GMRES(m) was formulated as a control problem for which m is the control variable modified to be modified at each cycle by a discrete proportional-derivative controller. The controller has the capacity to augment or deflate the dimension of the Krylov subspace if any convergence problem is detected. In [3] a combination of both feedback control of m and subspace enrichment with a fixed amount of harmonic Ritz vectors was added.

[1]gustavoespinola1989@gmail.com
[2]jccabral19@gmail.com
[3]torres.pedrozpk@gmail.com
[4]cschaer@pol.una.py

3 Conclusion

The results of numerical experiments with three adaptive Restarted GMRES algorithms, with the parameters m, d, l as control variables are compared. A discussion of the appropriate variation of subspaces is performed for identifying when to use either an acceleration or an overcoming stagnation strategy in contraposition to the standard GMRES(m).

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