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Numerical studies of unidimensional peridynamic problems

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1 Abstract

The peridynamic theory is an extension of the classical continuum mechanics theory and is based on long-range interaction forces between material points separated by finite distances, which are usually smaller than a parameter l called horizon. These forces depend upon the relative displacements and positions between material points within a body rather than the spatial derivatives of displacements, which are used in classical constitutive relations, allowing modeling of discontinuities such as cracks. This work concerns analysis of 1D problems within the context of peridynamics through numerical methods, with the purpose of gaining understanding in future analysis of 2D and 3D problems.

The peridynamic equilibrium problem of an infinite linearly elastic bar consists of finding the longitudinal displacement field $u : (-\infty, \infty) \to \mathbb{R}$ that satisfies the integral equation

$$\int_{-\infty}^{\infty} C(\xi)\left(u(x-\xi)+u(x)\right)d\xi + b(x) = 0, \qquad \forall x \in (-\infty, \infty), \tag{1}$$

where $C : (-\infty, \infty) \to \mathbb{R}$ is the micromodulus of a peridynamic material and $b : (-\infty, \infty) \to \mathbb{R}$ is the longitudinal body force field. For comparison purposes, we also consider the equilibrium problem of an infinite linearly elastic bar in the context of the classical theory, which consists of finding $u : (-\infty, \infty) \to \mathbb{R}$ that satisfies the differential equation

$$E\frac{d^2u}{dx^2} + b(x) = 0, \qquad \forall x \in (-\infty, \infty), \tag{2}$$

where E is the Young's modulus. Considering that this bar is subjected to a dipole of unitary body force such that $b(x) = -\delta(x+a) + \delta(x-a)$, where δ is the Dirac delta function, Mikata [2] uses Fourier transform to obtain a solution u of (1). Bobaru et al. [1] uses a numerical technique to investigate convergence of approximate solutions of (1).

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In this work, we investigate problems with micromoduli given by

$$C_1(\xi) = \begin{cases} 9.3547\frac{E}{l^3\sqrt{\pi}}e^{-\left(\frac{\xi}{l}\right)^2}, & |\xi| \leq l \\ 0, & |\xi| > l \end{cases}, \quad C_2(\xi) = \frac{4E}{l^3\sqrt{\pi}}e^{-\left(\frac{\xi}{l}\right)^2}, -\infty < \xi < \infty. \qquad (3)$$

These micromoduli were not considered in the numerical investigation conducted by Bobaru et al. [1]. The micromodulus C_2 allows a material point to interact with points at a distance larger than the horizon. Fig. 1 shows the absolute value of the difference between the peridynamic approximate solution u_h of (1) and the classical exact solution u of (2) vs the horizon l considering the micromoduli given in (3). It is clear that the PD solution converges to the classical one as the horizon decreases.

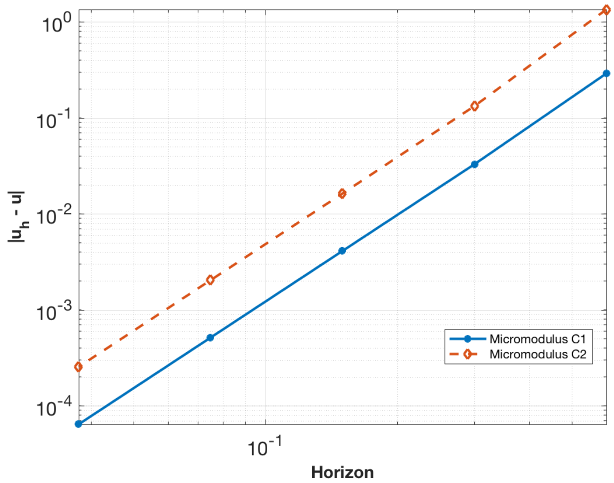

Figure 1: Error in L_2 norm between peridynamic approximate and classical exact solutions vs the horizon for the micromoduli in (3).

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References

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