STABLE LOW-ALTITUDE POLAR ORBITS AROUND EUROPA

JOSUÉ CARDOSO DOS SANTOS*, JEAN PAULO DOS SANTOS CARVALHO†, RODOLPHO VILHENA DE MORAES†

*333 Ariberto Pereira da Cunha Ave, Portal das Colinas
Engineering College of Guaratiguetá (FEG)-UNESP
Guaratinguetá, São Paulo State, Brazil

†330 Talim St, Jardim das Nações
Institute of Science and Technology (ICT)-UNIFESP
São José dos Campos, São Paulo State, Brazil

Emails: josuesantosunesp@gmail.com, jeanfeg@gmail.com, rodolpho.vilhena@gmail.com

Abstract— In this paper, a search for low-altitude orbits around Europa is performed. An emphasis is given in frozen orbits. These orbits are used in the planning of aerospace activities to be conducted around this planetary satellite, with respect to the stability of orbits of artificial satellites. The study considers orbits of artificial satellites around Europa under the influence of the third-body perturbation (the gravitational attraction of Jupiter) and the polygenic perturbations. These last ones occur due to forces such as the non-uniform distribution of mass (*J2* and *J3*) of the main (central) body. A simplified dynamic model for these perturbations is used. The Lagrange planetary equations, which compose a system of nonlinear differential equations, are used to describe the orbital motion of the artificial satellite around Europa. The equations of motion are developed in closed form to avoid expansions in power series. Low-altitude polar orbits and frozen orbits with long lifetimes were found. This study can be used for planning of future space missions which seek to visit Europa and other planetary satellites.

Keywords— Astrodynamics and Orbital Mechanics, Frozen orbits, Planetary Satellite, Europa, Single-averaged method.

Resumo— Neste trabalho, uma busca por órbitas de baixa altitude em torno de Europa é realizada. Uma ênfase é dada em órbitas congeladas. Estas órbitas serão utilizadas no planejamento de atividades aeroespaciais a serem conduzidas em torno deste satélite planetário, no que diz respeito à estabilidade de órbitas de satélites artificiais. O estudo leva em consideração órbitas de satélites artificiais em torno de Europa sob a influência da perturbação de terceiro corpo (a atração gravitacional de Júpiter) e das perturbações devidas às forças poligênicas. Estas últimas ocorrem devido às forças tais como a distribuição não uniforme de massa (*J2* e *J3*) do corpo principal (central). Aqui é utilizado um modelo dinâmico simplificado para essas perturbações. As equações do movimento são desenvolvidas em forma fechada para evitar expansões em séries de potência. Órbitas polares de baixa altitude e órbitas congeladas com tempos de vida longos foram encontradas. Este estudo pode ser utilizado para o planejamento de futuras missões espaciais que pretendem visitar Europa e outros satélites planetários.

Palavras-chave— Astrodinâmica e Mecânica Orbital, Órbitas Congeladas, Satélite Planetário, Europa, Método da média simples.

1 Introdution

Europa is one of the four largest moons of Jupiter. It is one of the planetary satellites of greater interest at the present moment among the scientific community. In the last years are being planned some missions to visit Europa. As an example, we can cite the Jupiter Europa Orbiter (JEO, NASA) and the Jupiter IcyMoon Explorer (JUICE, ESA). Several papers have contributed to a better understanding of the dynamics involved in this type of mission (see for example Scheeres et al., 2001; Prado, 2003; Paskowitz & Scheeres, 2005a, 2005b, 2006; Lara & Russel, 2006, 2007; Domingos et al., 2008; Carvalho et al., 2010, 2012a, 2012b; Liu et al., 2012). The study performed in this paper is related to the orbital motion of artificial satellites around Europa considering polygenic perturbations due to its non-spherical shape and the third-body perturbation due to Jupiter. The Lagrange planetary equations, which compose a system of nonlinear differential equations, are used to describe the orbital motion of the artificial satellite. The simplified dynamic model considers the effects caused by non-uniform distribution of mass of Europa (*J2* and *J3*) and the gravitational attraction of third body (in this paper this one is Jupiter and it is considered in a circular orbit). In this paper we are searching for frozen orbits to be used for aerospace tasks as regards the stability of artificial satellite orbits around Europa. Some frozen orbits were found, analysed and the results are presented in the Sect. 5. This paper is organized as follows. In Sect.2, the system of motion equations (Lagrange planetary equations) is presented. Next, the Sect.3 follows presenting the model for the third-body perturbation without the elimination of the Jupiter's mean anomaly. After that, in Sect.4, are presented the perturbations due to the non-sphericity of the central body. Finally, in Sect.5 are presented results relative to the disturbing potential effect over some orbital elements of the spacecracft such as eccentricity, argument of the periapsis, inclination and longitude of the ascending node. These results were obtained by performing numerical simulations using the software Maple.

2 Equations of motion

In this section are presented the Lagrange planetary equations for describing the spacecraft perturbed orbital motion. This motion is described by six variables called orbital elements, which ones are denoted by: a (semimajor axis), e (eccentricity), i (inclination), h (longitude of the ascending node), g (argument of the periapsis) and l (mean anomaly). These equations describe the variance with respect to the time of the spacecraft orbital elements and are given by (Kovalewsky,1967)

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial R}{\partial l} \tag{1}$$

$$\frac{de}{dt} = \frac{-\sqrt{1-e^2}}{na^2e}\frac{\partial R}{\partial g} + \frac{1-e^2}{na^2e}\frac{\partial R}{\partial l} \tag{2}$$

$$\frac{di}{dt} = \frac{1}{na^2\sqrt{1-e^2}\sin(i)}\left(\frac{\partial R}{\partial g}\cos(i) - \frac{\partial R}{\partial h}\right) \tag{3}$$

$$\frac{dh}{dt} = \frac{1}{na^2\sqrt{1-e^2}\sin(i)}\frac{\partial R}{\partial i} \tag{4}$$

$$\frac{dg}{dt} = \frac{\sqrt{1-e^2}}{na^2e}\frac{\partial R}{\partial e} - \frac{\cos(i)}{na^2\sqrt{1-e^2}\sin(i)}\frac{\partial R}{\partial i} \tag{5}$$

3 Third-Body Perturbation

The term due to third-body perturbation (Jupiter) in a circular orbit is the long-period disturbing potential (Carvalho et al., 2013)

$$R_2 = \frac{15}{32}{n_J}^2a^2(A+B+CD+E+F) \tag{6}$$

where

$$A = e^2(\cos(i)+1)^2\cos(2g+2h-2l_J) \tag{7}$$

$$B = e^2(\cos(i)-1)^2\cos(2g-2h+2l_J) \tag{8}$$

$$C = -\frac{6}{5}(\cos(i)-1)(e^2+\frac{2}{3}) \tag{9}$$

$$D = (\cos(i)+1)\cos(2h-2_J) \tag{10}$$

$$E = 2e^2(1-\cos^2(i))\cos(2g) \tag{11}$$

$$F = \frac{6}{5}(e^2+\frac{2}{3})(-\frac{1}{3}+\cos^2(i)) \tag{12}$$

Table 1: Numerical values for J_2 and J_3

J_2	4.355x10^{-4}
J_3	1.3784x10^{-4}

where the single-average was applied only for the elimination of the short-period terms. It is important highlight that the mean anomaly (l_J) of the disturbing body was not eliminated as it was performed in previous papers (Scheeres et al., 2001; Prado, 2003; Liu et al., 2012). The orbit of Jupiter is considered circular and fixed in space, so the Jupiter's mean anomaly was not eliminated. Numerical values used can be seen at JPL (http://naif.jpl.nasa.gov/naif/index.html).

This disturbing potential were developed considering Jupiter and Europa on the same plane. Thus, the existence of a small inclination between the orbit of Jupiter around Europa was neglected (see Liu et al., 2012).

4 Polygenic Perturbations

For the perturbations caused by the non-uniform distribuition of mass of the main body (Europa) was taken into account its non-sphericity as performed in previous papers like Scheeres et al. (2001), Paskowitz & Scheeres (2006), Lara & Russel (2007) and Carvalho et al (2012a, 2012b). According to Carvalho et al. (2013), the zonal perturbation due to the oblateness of the main body is given by

$$R_{J2} = -\frac{1}{4}\frac{\epsilon_1 n^2(-2+3\sin^2(i))}{(1-e^2)^{3/2}} \tag{13}$$

Also according to (Carvalho et al., 2013), the zonal perturbation due to the pear-shaped is given by

$$R_{J3} = -\frac{3}{8}\frac{e\epsilon_2 n^2\sin(i)(-4+5\sin^2(i))\sin(g)}{(1-e^2)^{5/2}a} \tag{14}$$

where n is the mean motion of the artificial satellite and R_M is the mean ratio of the main body. The parameters ϵ_1 and ϵ_2 are given by $\epsilon_1 = J_2{R_M}^2$ and $\epsilon_2 = J_3{R_M}^3$.

In this paper are considered only the harmonic coefficients *J2* and *J3*. The numerical values for these coefficients are presented in Table 1 (Lara & Russell, 2007).

5 Results

The search for polar orbits around Europa is due to the fact that this type of orbit allows a better coverage of the surface of the main body (Europa) than other types of orbits. The orbits around Europa have, generally, around 160 days of duration (Paskowitz & Scheeres, 2005a; 2005b), which

DOI: 10.5540/03.2013.001.01.0189

means that they have short duration. Carvalho et al. (2013) found orbits with periods longer than 300 days. Thus, a study with a duration of around 500 days covers a time long enough for several types of missions.

The disturbing potential taking into account the third-body perturbation (R_2) and the non-uniform distribution of mass (J_2 and J_3) can be written as

$$R = R_2 + R_{J2} + R_{J3} \tag{15}$$

This disturbing potential is replacing in the system of Lagrange planetary equations (Eqs. 1-5) and these are numerically integrated using the software Maple. The obtained results from the simulations are presented as follow.

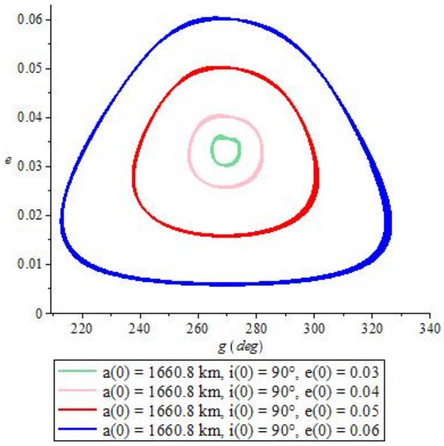

Figure 1: Diagram e vs g. Initial conditions: $h = 90°$ and $g = 270°$. Time evolution of 500 days.

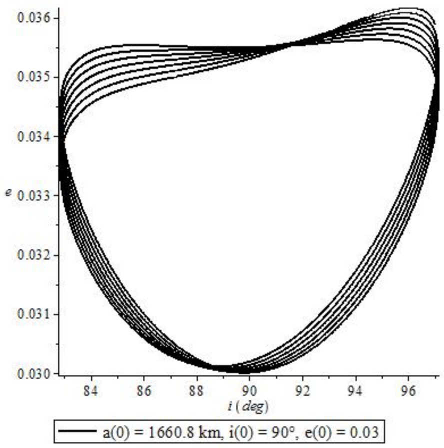

Figure 2: Diagram e vs i. Initial conditions: $h = 90°$ and $g = 270°$. Time evolution of 500 days.

Frozen orbits were found by performing this process that librates around equilibrium points for $g = 270°$, $h = 90°$, $i = 90°$ and 1660.8 km $\leq a \leq 1850$ km. Fig.1, Fig.5 and Fig.9 show the behavior of a polar orbit around Europa using the analytical equations. The regions of libration are showed

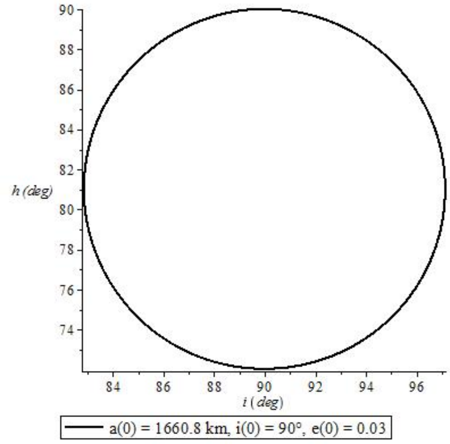

Figure 3: Diagram h vs i. Initial conditions: $h = 90°$ and $g = 270°$. Time evolution of 500 days.

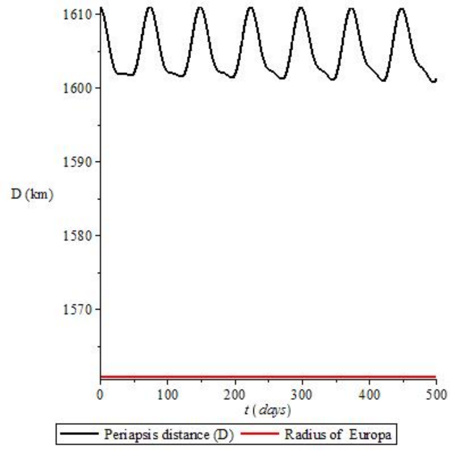

Figure 4: Diagram periapsis distance vs time. Initial conditions: $h = 90°$ and $g = 270°$. Time evolution of 500 days.

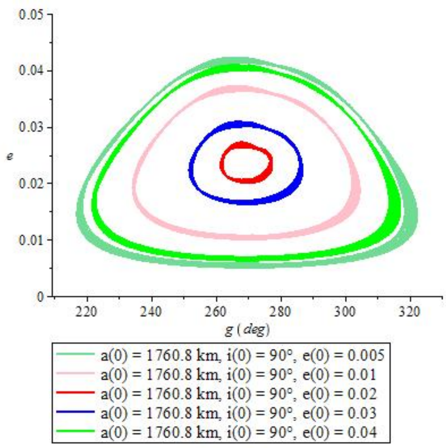

Figure 5: Diagram e vs g. Initial conditions: $h = 90°$ and $g = 270°$. Time evolution of 500 days.

for many values of eccentricity in the diagrams e vs g (Fig.1, Fig.5 and Fig.9). The Figs.1-4 show the results for orbits with semimajor axis equal to 1660.8 km (100 km of altitude). The Figs.5-8 show

DOI: 10.5540/03.2013.001.01.0189

the results for orbits with semimajor axis equal to 1760.8 km (200 km of altitude). The Figs.9-13 show the results for orbits with semimajor axis equal to 1850 km (289.2 km of altitude). We can see that the main difference between these initial conditions of semimajor axis are the amplitudes of libration. The diagrams following each one of the diagrams e vs g show analysis of the solution that has smaller amplitude of libration in these last ones. The case $e(0)$=0.02 (pink line in Fig.9) present a very interesting result because this solution has a very small amplitude of libration. For a better view of this solution, it is plotted alone in Fig.10. According the results obtained by the simulations, the set of initial conditions with smaller amplitude of libration is given by a = 1850 km, e = 0.02, $g = 270\,°$, $h = 90\,°$, $i = 90\,°$. (see Liu et al., 2012). The Fig.2, Fig.6 and Fig.11 show the diagrams e vs i. It is possible observe that when a = 1660.8 km, the eccentricity varies from 0.0200 to 0.0215, while when a = 1760.8 km, it ranged from 0.030 to 0.036 and when a = 1850 km, it ranged from 0.020 to 0.027.

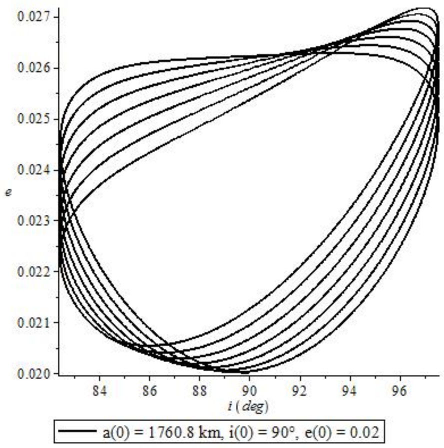

Figure 6: Diagram e vs i. Initial conditions: $h = 90\,°$ and $g = 270\,°$. Time evolution of 500 days.

The diagrams h vs i appear in Fig.3, Fig.7 and Fig. 12 and as we can see, they produced practically the same results (the inclination ranged from about $72\,°$ to $90\,°$). D represents the distance of the periapis position away from the center of Europa. The expression for D is $D = a(1\text{-}e)$. Finally, the Fig. 4, Fig.8 and Fig.13 present the diagrams D vs *time*. These diagrams show the time variation of the periapsis distance during 500 days. All the results produced low-altitude stable orbits. Note, specially, the Fig.4 and the Fig.13. In the first one, it is possible to observe periods where there is an aproximation of about 51 km from the Europa surface. It is a great result considering the possibility of performing more detailed studies of the Europa surface. The second one shows an orbit that has a very small amplitude of range in the periapsis distance. For this case (a = 1850

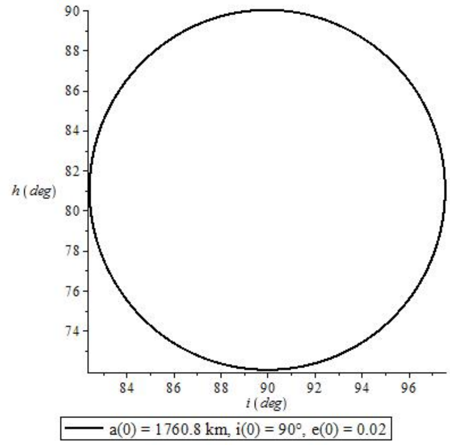

Figure 7: Diagram h vs i. Initial conditions: $h = 90\,°$ and $g = 270\,°$. Time evolution of 500 days.

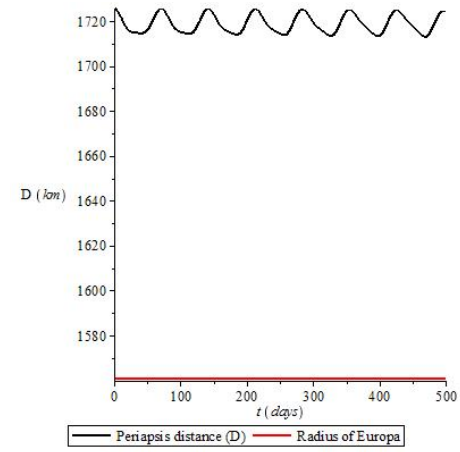

Figure 8: Diagram periapsis distance vs time. Initial conditions: $h = 90\,°$ and $g = 270\,°$. Time evolution of 500 days.

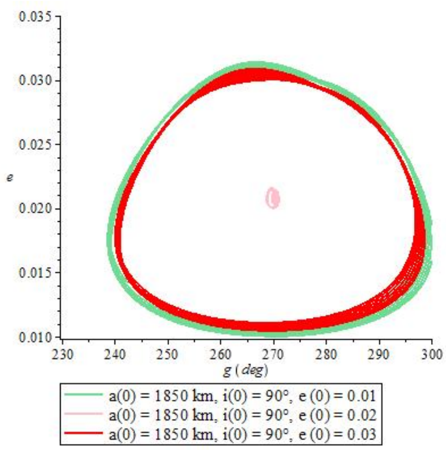

Figure 9: Diagram e vs g. Initial conditions: $h = 90\,°$ and $g = 270\,°$. Time evolution of 500 days.

km), all the orbital elements produced very small variations, sometimes considerable smaller than compared to other cases (a = 1660.8 km and a = 1760.8 km). This important characteristic may

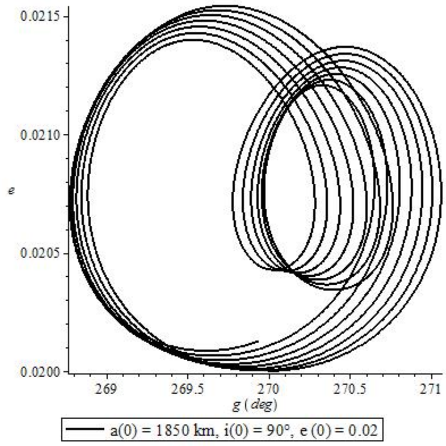

Figure 10: Diagram e vs g. Initial conditions: $h = 90\,°$ and $g = 270\,°$. Time evolution of 500 days.

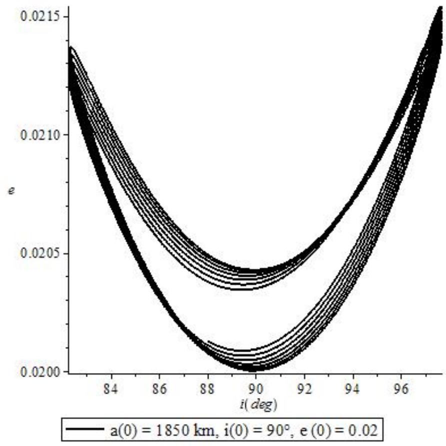

Figure 11: Diagram e vs i. Initial conditions: $h = 90\,°$ and $g = 270\,°$. Time evolution of 500 days.

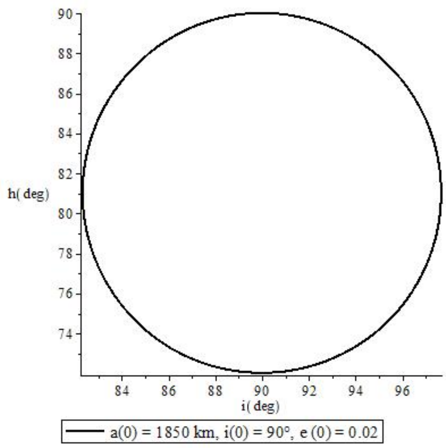

Figure 12: Diagram h vs i. Initial conditions: $h = 90\,°$ and $g = 270\,°$. Time evolution of 500 days.

be considered in future missions to Europa. The greater importance of frozen orbits is that they require small amounts of fuel for orbital maintenance. Thus, there is a lower cost to keep the artificial satellite in specific orbits. The specific

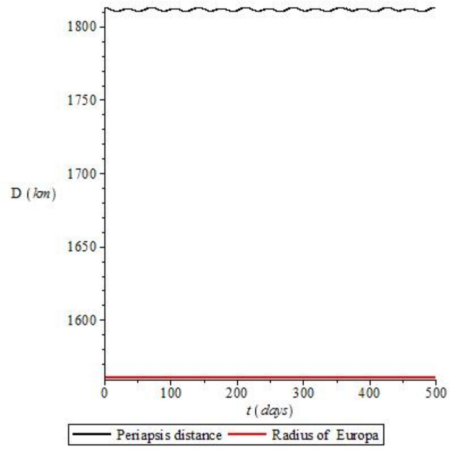

Figure 13: Diagram periapsis distance vs time. Initial conditions: $h = 90\,°$ and $g = 270\,°$. Time evolution of 500 days.

ones are those that pass at the same altitude for a given latitude, benefiting the users with this regularity. It means that this type of orbit maintains a value of altitude almost constant over any point on the main body surface. Since a satellite in low-altitude polar orbit (about 100 km) around Europa collides in a short time period (Paskowitz & Scheeres, 2005a; 2005b), it is important to look for any polar orbits in which the artificial satellite has a longer lifetime. Indeed, we found a region of initial semimajor axis where the polar orbits survive longer than that found in the literature (Paskowitz & Scheeres, 2005a; 2005b). A semimajor axis of 1660.8 km is a very good location for an artificial satellite because it implies an altitude of 100 km from the Europa surface. The results show that in this case, the orbit remains for periods longer than 500 days (see Fig.4). A semimajor axis of 1850 km is also a very good location for an artificial satellite because it implies a low-altitude orbit of 289.2 km from the Europa surface and a very small variations in the orbital elements. Thus, this result gives a solution with lower cost of maintenance. In this case, the orbit also remains for a period of time longer than 500 days (see Fig.13), just like the other ones. The orbits found in this research are near circular frozen polar orbits whose semimajor axis are in the interval between 1660.8 km and 1850 km. Carvalho et al. (2012a) performed a study taking into account the non-uniform distribution of mass of the planetary satellite (*J2*, *J3* and *C22*) and the gravitational attraction of a third body (Jupiter) which one was considered in a elliptic orbit. For the orbits analyzed in their study, the lifetimes were estimated by the single-averaged method and by the double-one. In the same study, comparisons between the results obtained by these two methods were also performed. The results showed that the single-averaged method produced more realistic results than the other one. Finally, they found unstable polar orbits with at least 200 days of lifetime. In our paper, we found stable frozen polar orbits with lifetimes longer than 500 days. It is important to emphasize that the values of altitude found in this paper are lower than the ones presented in Carvalho et al. (2012a; 2013).

6 Conclusions

Low orbits are subject of great interest of the scientific community for its proximity to the main body (Europa), which allows more detailed investigations such as the study of the planetary satellite surface and its gravitational field. In some analysis, for the case of a lunar orbit, it is possible to neglect the third-body perturbation (Earth). But in the case of Europa, the disturbing body (Jupiter) cannot be neglected in the dynamics because of the its strong perturbation. In this paper, the disturbing body (Jupiter) was considered in a circular orbit. In the disturbing potential were considered the perturbations due to the third-body ($R2$) and due to the non-uniform distribution of mass (due to the harmonic coefficients $J2$, $J3$). The single-averaged method was used in the development of the motion equations which means that the Jupiter's mean anomaly was not eliminated. Simulations were performed for some low-altitude polar orbits using the software Maple and some results were presented for three of them (100 km, 200 km and 289.2 km). The orbits with altitude of 100 km and 200 km are planned for the spacecraft Jupiter Europa Orbiter (JEO, NASA) and good results with small amplitudes of libration were obtained for these ones. Similar good results were obtained for other values of low-altitudes. It represents a very good result because, generally, these orbits are very unstable. These results can be used for future missions to Europa and similar studies can be developed for other planetary satellites. For future studies the eccentricity of Jupiter, the harmonic term $C22$ due to the non-sphericity of Europa and the perturbation caused by the Jupiter's planetary satellite Ganimede will be considered.

Acknowledgments

The authors wish to acknowledge the support from UNESP, UNIFESP and FAPESP (proces. n^o 2011/05671-5, 2012/21023-6 and 2012/12539-9). Special thanks to God for all the opportunities given during the development of this paper.

References

Carvalho, J. P. S.; Vilhena de Moraes, R.; Prado, A. F. B. A. *Some orbital characteristics of lunar artificial satellites.* Cel. Mec. & Dyn. Astr., v.108, no.4, pp.371-388, 2010. DOI: 10.1007/s10569-010-9310-6

Carvalho, J. P. S.; Elipe, A.; Vilhena de Moraes, R.; Prado, A. F. B. A. *Low-altitude, near-polar and near-circular orbits around Europa.* Adv. Spac. Research., v.49, no. 5, pp.994-1006, 2012a.

Carvalho, J. P. S.; Vilhena de Moraes, R.; Prado, A. F. B. A. *Frozen orbits around Europa.* Inter. J. of Bifurc. and Chaos in Appl. Sci. and Engin., v.22, no.10, Art. ID 1250240, 2012b. DOI: 10.1016/j.asr.2011.11.036

Carvalho, J. P. S.; Vilhena de Moraes, R.; Prado, A. F. B. A. *Dynamics of Artificial Satellites around Europa.* Math. Prob. Eng. 2013, Art. ID 182079, 2013.

Domingos, R. C.; Vilhena de Moraes, R.; Prado, A. F. B. A. *Third-body perturbation in the case of elliptic orbits for the disturbing body.* Math. Prob. Eng. 2008, Art. ID 763654, 2008.

Giacaglia, G. E. O.; Murphy, J. ; Felsentreger, T. *A semianalytic theory for the motion of a lunar satellite.* Celestial Mechanics, v.3, no.1, pp.3-66, 1970. DOI: 10.1007/BF01230432

Kovalevsky, J. *Introduction to Celestial Mechan-ics*, Bur. des Longi., Paris, France, 1967. DOI: 10.1007/978-94-011-7548-7

Lara, M. and Russell, R. *On the design of a science orbit about Europa* in Space Flight Mechanics Meeting, paper 2006 AAS/AIAA, Tampa, Fla, USA, January 2006.

Lara, M. and Russell, R. *Computation of a science orbit about Europa* J. Guid. Control and Dyn., v.30, no.1, pp.259-263, 2007. DOI: 10.2514/1.22493

Liu, X.; Baoyin, H.; Ma, X. *Long-term perturbations due to a disturbing body in elliptic inclined orbit.* Astr. and Space Sci., v.339, no.2, pp.295-304, 2012.

Paskowitz, M. E. and Scheeres, D. J. *Orbit mechanics about planetary satellites including higher order gravity fields* in Space Flight Mech. Meet., paper AAS 2005-190, Copper Mountain, Colo, USA, 2005a.

Paskowitz, M. E. and Scheeres, D. J. *Transient behavior of planetary satellites including higher order gravity fields* in Proc. of the Astrodyn. Spec. Conf., paper AAS 2005-358, Lake Tahoe, Calif, USA, 2005b.

Paskowitz, M. E. and Scheeres, D. J. *Design of science orbits about planetary satellites: application to Europa.* J. Guid. Control and Dyn., v.29, no.5, pp.1147-1158, 2006. DOI: 10.2514/1.19464

Prado, A. F. B. A. *Third-body perturbation in orbits around natural satellites.* J. Guid. Control and Dyn.,v.26, no.1, pp.33-40, 2003. DOI: 10.2514/2.5042

Scheeres, D. J.; Guman, M. D.; Villac, B. F. *Stability analysis of planetary satellite orbiters: application to the Europa orbiter.* J. Guid. Control and Dyn., v.24, no.4, pp.778-787, 2001. DOI: 10.2514/2.4778

