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Analytical Model Study for the Pollutants Dispersion with Coherent Structures Presence

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1 Introduction

Since the beginning of the industrial revolution there has been an effort by the scientific community to understand and describe the phenomenon of dispersion of pollutants in order to develop tools to monitor the impact caused by emissions.

The present work is an analysis of the behavior of the distributions generated by an analytical solution of the advection diffusion equation [1] for pollutants dispersion. This model is unique in that it realistically describes aspects of turbulent phenomena neglected by other analytic models. Previous studies have defined as coherent structures some turbulent characteristics such as vortices, fluctuations in pollutant density, and swirls. Formally, coherent structures are defined through a phase difference property that does not exist in the advection diffusion equation [2]. Modifications were applied in the Fick closure of the equation where a phase difference was included through the insertion of a complex diffusive coefficient.

The parameters linked to the inclusion of the phase need to be analyzed and understood so new possibilities can be aggregated enabling a proper parameterization of the deterministic model with stochastic effects.

2 The Model

The diffusion advection equation is solved analytically where the diffusive terms are constant in K_x and K_y. Now the coefficient K_z is complex having a trigonometric profile in the real part and constant in the imaginary part $K_z = K_{za} \sin\left(\frac{z\pi}{L_z}\right) + iK_{zb}$, where the term L_z $[m]$ refers to the vertical domain, K_{za} and K_{zb} are constant values. Separation

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of variables followed by Sturm-Liouville are used to solve the problem associated with the dimensions y and z. For the dimension x and the temporal term t the technique of transform of Laplace is applied. Thus the solution is given by multiplication, $C(x, y, z, t) = C(x, t)C(y)C(z)$. This is the complex solution of the model, the concentration distributiom is obtained multiplying this solution by its conjugate complex [3]. This analytical model solution has two very important characteristics to describe the phenomenon of pollutants dispersion. One is a semi-positive property of distributions and the other introduces turbulent structures compatible with coherent structures through the sesquilinear form of the model. By evaluating the behavior of the distributions generated by the model from the variation of the parameters of K_z in Figure 1, it is seen that K_{za} is only responsible for the dispersion speed. The paramter K_{zb}, besides being related to the dispersion, causes changes in the patterns of the coherent structures.

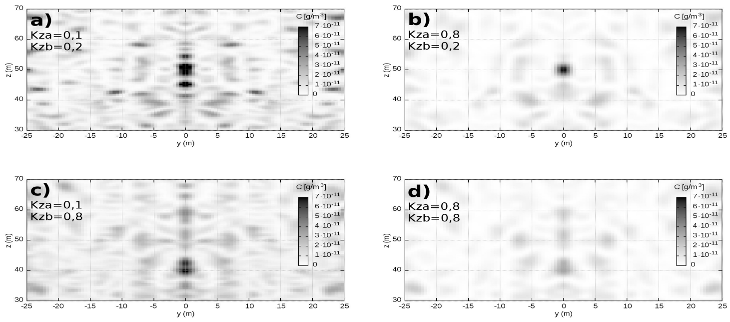

Figure 1: Maps of the distribution in $C(y, z)$ $[g/m^3]$ showing the dispersion behavior of a source in $y = 0$ $[m]$ and $z = 50$ $[m]$. The parameters of the complex coefficient K_z are varied. The real part K_{za} only shows a dispersion of the pollutant in (a and b) or (c and d). The imaginary part K_{zb} shows modification of turbulent structures for each value inserted in (a and c) or (b and d).

This behavior gives indications that there is a correlation between the complex part and the amount of convectivity being generated by the medium. This is a result that guides the work to find physical relationships to obtain a parameterization of the model to represent the most diverse atmospheric scenarios.

References

[1] S. P. Arya, Air Pollution Meteorology and Dispersion, *Oxford University Press*, New York, USA, (1999).

[2] D. L. Gisch and B. E. J. Bodmann and M. T. Vilhena, Two reasons why pollution dispersion modelling needs sesquilinear forms.,*Integral Methods in Science and Engineering*, Springer International Publishing,chapter 22,2015

[3] D. L. Gisch and B. E J Bodmann and M. T. M. B. Vilhena, A Genuine Solution of the Diffusion Advection Equation Sesquilinear Way to Multi-Source Problem, 6:160-163, *AJEE*, 2016, DOI:10.5923/s.ajee.201601.23

