Proceeding Series of the Brazilian Society of Applied and Computational Mathematics, v. 6, n. 1, 2018.

Trabalho apresentado no XXXVII CNMAC, S.J. dos Campos - SP, 2017.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Equilibrium Conditions for Tethered Satellite Constellations

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1 Introduction

The space tether systems problem has been studied in several published articles. Beletsky and Levin [1] begins by setting the scene for tethers in space. They summarized possible applications and also discussed fact and fiction, analyzing clearly the main parameters and applications for tethers system as the material density, the effective forces, orbital dynamics, mechanics models, attitude and possible disturbances for flexible tethers. This work is a study for a tether system with six bodies connected by a cable, which moves your center of mass in plane and around the main body. The equilibrium analysis is made for m_1 and m_2 plane and out plane movement. Based on the papers mentioned above and in particular the work [1] - [3].

2 Mathematical model

For the coordinates system the components of center of mass position vector (x_0, y_0, z_0) and components of point mass i position vector (x_i, y_i, z_i) for six bodies. The Lagrange Motion Equations, second-order ordinary differential equations, which describe the motions mechanical systems under the action of forces, can be obtained from the Lagrangian of the system given by $L = T - V$. Being T the kinetic energy and V potential energy. The generalized coordinates are φ and ψ. The system is only under the gravity-gradient.
Defining φ or $l(lx, ly, lz)$ allows to control the system.It has been chosen to define φ and consecutively obtain the l behavior. The motion in the spherical coordinates, ψ, φ, ν and l, where lx, ly, lz represents the cable length, in the corresponding direction x, y and z, ν and φ in-plane and ψ out-plane rotation, being p the focal parameter, e the eccentricity and ν the true anomaly, the masses bodies (m_i) are equal and $\rho = \frac{p}{(1+e\cos(\nu))}$.

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$$
\begin{aligned}
&4(1+e\cos(\nu))\left(\varphi'+1\right)\cos^2(\psi)\left(lxlx'+lyly'\right)+4lzlz'\left(\varphi'+1\right)\sin^2(\psi)(1+e\cos(\nu))\\
&\quad+lz(\nu)^2\left(\sin^2(\psi)\left(2\varphi''(1+e\cos(\nu))-4e\sin(\nu)+3\sin(2\varphi)\right)\right.\\
&\quad+2\varphi'\left(\psi'\sin(2\psi)(1+e\cos(\nu))-2e\sin(\nu)\sin^2(\psi)\right)\\
&\quad\left.+2\psi'\sin(2\psi)(1+e\cos(\nu))\right)=\cos(\psi)\left(2\left(lx^2\right.\right.\\
&\qquad\left.+ly^2\right)\left(2\left(\varphi'+1\right)\left(\psi'\sin(\psi)(1+e\cos(\nu))+e\sin(\nu)\cos(\psi)\right)-\varphi''\cos(\psi)(1+e\cos(\nu))\right)\\
&\qquad\left.+3\left(ly^2-lx^2\right)\sin(2\varphi)\cos(\psi)\right)
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
&2\left(lx^2+ly^2+lz^2\right)\left((1+e\cos(\nu))\psi''-2e\sin(\nu)\psi'\right)\\
&\quad+\left(\varphi'+1\right)^2\sin(2\psi)(1+e\cos(\nu))\left(lx^2+ly^2-lz^2\right)\\
&\quad+4\psi'(1+e\cos(\nu))\left(lxlx'+lyly'+lzlz'\right)\\
&\quad+3\sin(2\psi)\left(lx^2\cos^2(\varphi)+ly^2\sin^2(\varphi)\right)-3lz^2\cos^2(\varphi)\sin(2\psi)=0
\end{aligned}
\tag{2}
$$

Mathematically, a point is in equilibrium when its speed and acceleration are equal to zero, then assume $\varphi'=\psi'=l'(lx,ly,lz)=0$ and $\varphi''=\psi''=l''=lx''=0$.

3 Conclusion

The system motion was obtained with the Lagrangian Formulation in a Central Gravitational Field, and the perturbations of motion are neglected. Laws of control are considered for the angle of system‘s rotation around the center of mass. The equilibrium of six mass points connected by a cable, orbiting a common center of mass, the center of mass fixed in line with the primary body was searched.

Acknowledgments

FAPESP - 16/15675-1, 2017/04643-4, CAPES BEX $10689-13-3$, INPE - Brazil, C-MAST - UBI - Portugal.

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