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Thermal Properties Identification Employing The Nelder-Mead and Hooke-Jeeves Algorithms Within The Topographical Global Optimization Technique

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The solution of the heat conduction equation in cartesian coordinates yields the temperature profile $T(x, y, z)$ for any given time t [1]. This is the direct problem. Now suppose that some - or all - of that properties and conditions are unknown, but experimental data of the temperature profile is available, one may then try to estimate the unknowns. This is the inverse problem.

Consider a system with a sandwich-like configuration, plate-resistance-plate. Two plates of bakelite are used with dimensions $L_x = 80$ mm, $L_y = 40$ mm, $L_z = 1.6$ mm each. A thin electrical resistance (39.9 Ω powered with 7.5 V) is employed in the middle of the superior half of the sandwich system.

Assuming that both plates are thermally thin and share the same heat flux provided by the resistance, and also that there is no temperature gradient in the y direction, one can use the Classical Lumped Analysis to describe this problem one dimension, as follows

$$k\frac{\partial^2 T(x,t)}{\partial x^2} - \frac{h_{ef}(x)[T(x,t) - T_\infty]}{L_z} + \frac{q''_{aq}(x)}{L_z} = \rho c_p \frac{\partial T(x,t)}{\partial t}. \tag{1}$$

with $T(x, 0) = T_\infty = 24$ °C, $h_{ef} = 17$ W/m^{2}K for $x < 40$ mm and 4 W/m^{2}K for $x > 40$ mm, $q''_{aq}(x) = 440$ W/m^2 for $x < 40$ mm and 0 for $x > 40$ mm and $\frac{\partial T}{\partial x} = 0$ in both boundaries.

To solve the direct problem, a implicitly formulated Finite Differences Method is employed. The thermal conductivity k and the specific heat c_p are considered unknowns. The inverse problem consists of an objective function $Q(c_p, k)$ given by the sum of the squared residues between the estimated (provided by solving the direct problem) and the

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experimentally measured temperature (provided by measurements with a infrared camera, similar to the one performed in [3]).

Hence, the inverse problem solution can be obtained by the minimization of this objective function. In this work, we propose the use of a clustering optimization method called Topographical Global Optimization (TGO) [2], which can be briefly described by three steps: **1** - Sampling a closed search space with N uniformly distributed random points. **2** - Selecting points with lower evaluations of objective function with respect to K-neighbors. **3** - Setting those selected points as initial solution for a local optimization method.

TGO was initialized with $N = 200$ points (generated by the Mersenne Twister point generator) and $K = 40$ neighbors. For the third step of TGO, two methods were tested: Nelder-Mead (NM) and Hooke-Jeeves (HJ). For sake of comparison, both methods have similar stopping criteria and were also initialized with the same set of initial solution.

In Table 1, "NFE"refers to "Number of Function Evaluations". It's noticeable that the NM as third step yields a much smaller NFE, therefore lower computational cost, making this a promising approach. Research must continue to investigate the perfomance of others local optimization algorithms as well as others points generators.

Table 1: Results obtained with N=200 e K=40. (All units in SI)

	Nelder-Mead				Hooke-Jeeves			
Exec.	c_p	k	Q	NFE	c_p	k	Q	NFE
#1	3395.89	0.377	490.008	396	3412.75	0.378	490.474	1277
#2	3395.89	0.377	490.008	316	3399.68	0.377	490.032	1330
#3	3395.89	0.377	490.008	375	3391.19	0.377	490.045	1013
#4	3395.89	0.377	490.008	324	3380.11	0.377	490.419	1044
#5	3395.89	0.377	490.008	332	3383.01	0.377	490.281	1306
Avg.	3395.89	0.377	490.008	348.6	3393.35	0.377	490.25	1194
σ	(10^{-4})	(10^{-8})	(10^{-10})	34.95	13.26	(10^{-4})	0.20	152.6

References

[1] M.N. Ozisik. *Heat Conduction.* John Wiley & Sons, New York, 1993.

[2] A. Törn and S. Viitanen. Topographical global optimization. *Recent Advances in Global Optimization*, Princeton University Press, Princeton, 384:398, 1992.

[3] D.C. Knupp, C.P. Naveira-Cotta, J.V.C. Ayres and R.M. Cotta. Theoretical-experimental analysis of heat transfer in nonhomogeneous solids via improved lumped formulation, integral transforms and infrared thermography. *Int. J. Therm. Sci.*, 62:71–84, 2012.