Proceeding Series of the Brazilian Society of Applied and Computational Mathematics, v. 6, n. 1, 2018.

Trabalho apresentado no XXXVII CNMAC, S.J. dos Campos - SP, 2017.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Role of parameters in the stochastic dynamics of a stick-slip oscillator

Roberta Lima[1]
Mechanical Engineering Department, PUC-Rio, Rio de Janeiro, RJ
Rubens Sampaio[2]
Mechanical Engineering Department, PUC-Rio, Rio de Janeiro, RJ

Abstract. In this paper a parametric analysis of a sample of responses of a dry-friction oscillator is performed in order to construct a statistical model. The system consists of a simple oscillator moving on a base with a rough surface. Due to this roughness, the mass is subject to a dry-frictional force modeled as a Coulomb friction. It is considered that the base has an imposed stochastic bang-bang motion which excites the system in a stochastic way and, induces stochastic stick-slip oscillations. The base velocity is modeled by a Poisson process for which a probabilistic model is fully specified. The system response is composed by a random sequence alternating stick and slip-modes. With realizations of the system, a statistical model is constructed for this sequence. Statistics and histograms of the random variables which characterize the stick-slip process are estimated. The objective of the paper is to analyze how these estimated statistics and histograms vary with the system parameters, i.e., to make a parametric analysis of the statistical model of the stick-slip process.

Keywords. Stick-slip, Dry-friction oscillator, Stick duration, Statistical model, Parametric analysis

1 Introduction

Dry-friction appears in several situations, as in drilling process and in mechanical gear systems. Despite the great number of papers in the area, few of them address the problem with a stochastic approach. The majority of the references only uses a deterministic approach. They do not discuss or quantify the uncertainties that are involved in the dynamics, although the dry friction force itself presents an inherent random behavior [1]. Therefore, a stochastic approach is the ideal way to address problems with dry-friction.

In this paper, we analyze the dynamics of a dry-friction oscillator which moves over a base with a rough surface. The base has an imposed stochastic bang-bang motion which excites the system in a stochastic way. The non-smooth behavior of the dry-friction force [2] associated with the non-smooth stochastic base motion induces in the system stochastic stick-slip oscillations. The system response is composed by a random sequence alternating stick and slip-modes. To characterize it, we construct a statistical model, in which the

[1]robertalima@puc-rio.br
[2]rsampaio@puc-rio.br

variables of interest of the random sequence are modeled as random variables [3, 4]. To estimate statistics and histograms of the system responses, samples of the random sequence of stick and slip-modes are computed by the integration of the dynamic equation of the system using independent samples of the base motion. The objective of the paper is to analyze how the estimated statistics and histograms vary with the system parameters, i.e., to make a parametric analysis of the statistical model of the stick-slip process.

2 Dynamics of a stick-slip oscillator

The system analyzed is composed by a simple oscillator (mass-spring) moving on a rough surface, as shown in Fig. 1. The roughness induces a dry-frictional force between

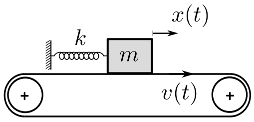


Figure 1: Stick-slip oscillator.

the mass and the base which is modeled as a Coulomb friction. Due to this friction model, the resulting motion of the mass can be characterized in two qualitatively different modes: the stick-mode (in which the mass and base have the same velocity during an open time interval) and the slip-mode, in which mass and base have different velocities. The position of the mass over the base is represented by x and its equation of motion is

$$m\,\ddot{x}(t) + k\,x(t) = f(t)\,, \tag{1}$$

where m is the mass, k is the spring stiffness and f is the frictional force between mass and base. During the slip-mode, it is assumed that $f(t) = mg\mu\,\mathrm{sgn}(v(t) - \dot{x}(t))$, where v is the base speed, g is the acceleration of gravity and μ is the friction coefficient (assumed to be constant). Thus, during the slip-mode, the value of the frictional force, f, is known. Its absolute value is equal to the maximum friction force, $f_{\max} = \mu mg$. During the stick-mode, $\dot{x}(t) = v(t)$, and thus equation of motion Eq. (1) can be rewritten as $m\dot{v} + k\,x(t) = f(t)$. The value of the frictional force during the stick-mode varies and it is confined to the interval $-f_{\max} \leq f \leq f_{\max}$. When the base speed is constant in time, during the stick-mode we have $-f_{\max} \leq k\,x(t) \leq f_{\max}$. Then, once in a stick-mode, the mass stays moving with the base until $x(t) = \frac{mg\mu}{k}$ in case of positive base velocity, or until $x(t) = -\frac{mg\mu}{k}$ in case of negative base velocity. Observe that during the stick-mode, the modulus of the elastic force increases up to the limit value $|f_{\max}|$. Remark that the duration of the stick-mode is bounded and its maximum value is $d_{\max} = 2\frac{mg\mu}{k\,v}$. For the slip one can make no prediction, in principle it can last forever.

3 Construction of a probabilistic model for the base motion

Considering that the dry-friction oscillator has an imposed stochastic bang-bang motion, we propose to model its velocity as a Poisson process, with constant rate λ, rep-

DOI: 10.5540/03.2018.006.01.0430

resented by $\mathcal{V}$. We consider that $\mathcal{V}$ is constant by parts and assumes only two values: $1,0$ m/s and $-1,0$ m/s. A realization of such stochastic process consists of point events in time which represents the instants in which occur changes of the velocity sign of the base motion. The parameter λ represents the expected value of number of changes per unit of time. As $\mathcal{V}$ is modeled as a Poisson process, the instants of change are given by random variables which can be ordered as $0 < Y_1 < Y_2 < Y_3 < \ldots$. From this sequence, it is possible to define the independent random variables $W_1 = Y_1$ and $W_j = Y_j - Y_{j-1}$, with $j \geq 2$. Each of them has exponential probability density function $p_{W_j}(t) = \mathbb{1}_{[0,\infty)}(t)\lambda\, e^{-\lambda t}$, with mean $1/\lambda$. The random variable W_j, with $j \geq 2$, indicates the waiting time between two consecutive change of the velocity sign of the base motion. Observe that a higher λ corresponds to a smaller average waiting time. The first change is at W_1, the second at $W_1 + W_2$, et cetera. Due to the bang-bang base motion, if the mass is in the stick-mode in the instant just before the discontinuity on the base velocity, it must be in the slip-mode in the instant just after the discontinuity. Thus, the stick is interrupted by the discontinuities on the base velocity, as if the dynamics were reinitialized; all previous information lost.

4 Construction of a statistical model of the stick-slip process

As it was assumed that the base motion is uncertain, the response of the stochastic stick-slip oscillator is a random process which presents a sequence alternating stick and slip-modes. Defined a time interval for analysis, the variables of interest in the dynamics are modeled as stochastic objects. We have two discrete random variables, which are the number of time intervals in which stick occurs (S_T) and the number of time intervals in which slip occurs (S_L). We have also discrete random processes, which are the instants at which the sticks begin ($T_1, \cdots, T_{S_T}$, where the subscripts $1, \cdots, S_T$ indicate the order that they occur), the duration of the sticks ($D_1, \cdots, D_{S_T}$), the instants at which the slips begin ($L_1, \cdots, L_{S_L}$) and, the duration of the sticks ($H_1, \cdots, H_{S_L}$). Figure 2 shows a sketch of the sequence of sticks and slips in the system response. Observe that we count the first slip just after the first stick.

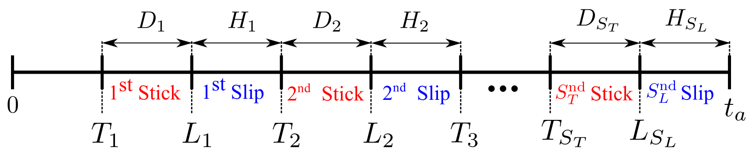


Figure 2: Sketch of the sequence of sticks and slips in the system response with $S_T = S_L$.

5 Strategy for parametric analysis

Two system parameters are considered in the parametric analysis of the statistical model of the stick-slip process. One of them is related to the probabilistic model of the

DOI: 10.5540/03.2018.006.01.0430

base motion λ and the other is the friction coefficient μ of the friction force. We performed numerical simulations combining different values of these parameters. To λ, 40 values were selected nonuniformly in the interval $[0.1,\ 30,0]$. To μ, 8 values were selected nonuniformly in the interval $[0.5,\ 7,0]$. For each combination of λ and μ, the dynamics equations were integrated 2,000 times using independent realizations of the base movement. A previous convergence study was developed to determine the acceptable number of realizations. In total, $640,000$ integrations were performed. In order to compute all these integrations, we adopted the strategy of parallelization of the simulations. Using a cluster composed of sixteen computers, as shown in Fig. 3, the computation time necessary to perform the $640,000$ integrations was approximately $55,5$ days. Without the parallelization, it would be need approximately $2,5$ years to compute the integrations, which is infeasible. For computation, duration t_a was chosen as 50 seconds. For the integration, it was used the function *ode45* of the *Matlab* software, which applies the Runge-Kutta 4th/5th-order method as time-integration scheme with a varying time-step algorithm. The maximal step size is equal to 10^{-4} seconds, and the relative and absolute tolerance are equal to 10^{-9}. The values of the parameters are $m = 1.0$ kg and $k = 4.0$ N/m. The initial conditions of the system were modeled as independent random variables, uniform distributed over $[-1,\ 1]$.

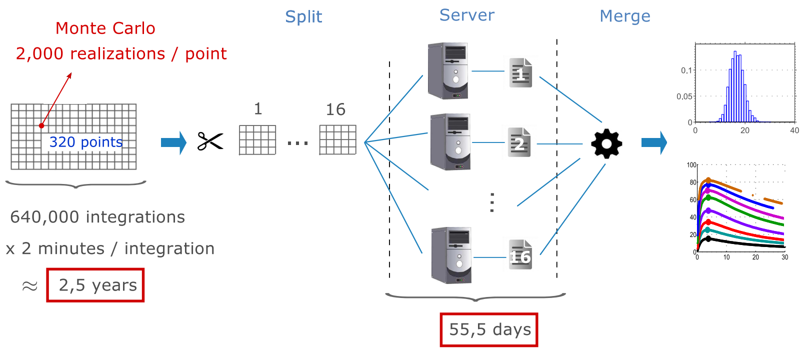


Figure 3: Parallelization of the simulations in the parametric analysis.

6 Influence of λ in the statistical model

First, we analyzed the influence of λ in the statistical model of the stick-slip process for a fixed value of μ. We took $\mu = 5.0$. In [3], it is possible to verify that for a fixed value of λ the duration of sticks are identically distributed. Then, we take $D_1, \cdots, D_{S_T}$ as identically distributed random variables and, we call them as D. To understand the influence of λ in D, we investigated the variation of the normalized histograms for different values λ between 0.1 1/s and 10.0 1/s. The results are shown in Fig. 4. For $\lambda = 0.1$ 1/s, we verify that the normalized histogram of D has one pick near to the maximum stick duration, which is $d_{\max} = 2\frac{mg\mu}{k\,v} = 2.5$ s. However as λ grows, this peak disappears, and the support of the normalized histogram is reduced. We conclude that the estimated mean of the stick duration decreases as λ grows. To quantify this decay, we plotted the estimated mean of the stick duration $\hat{\mu}_D$ as a function of λ, shown in Fig. 5.

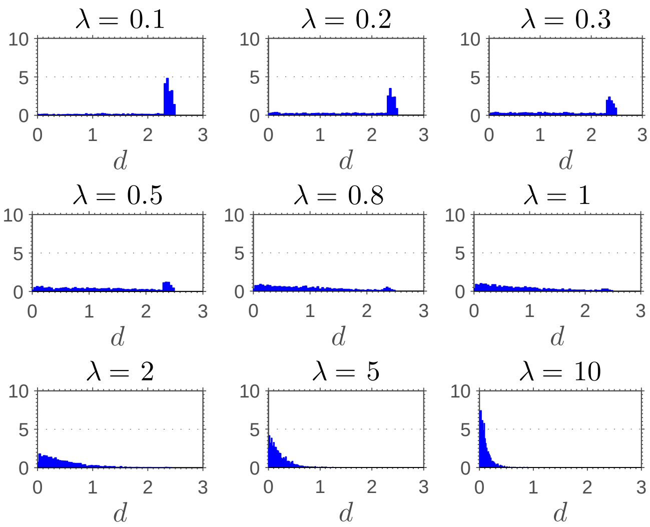


Figure 4: Normalized histograms constructed with $2,000$ samples of the duration of the first stick for six different values of λ.

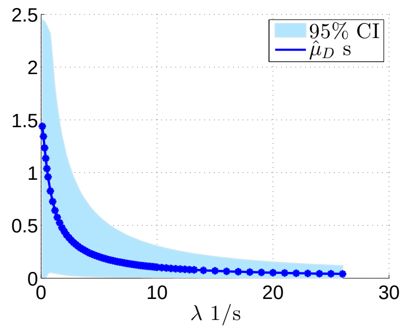
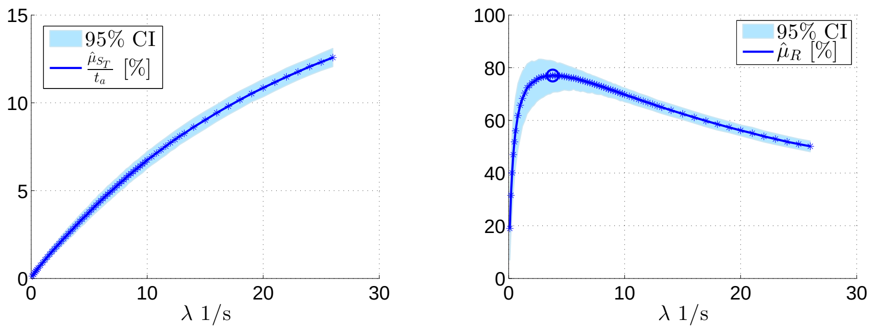


Figure 5: Estimated mean of the stick duration, $\hat{\mu}_D$, and 95 % confidence interval as a function of λ.

The graph of the estimated mean of the number of sticks divided by the duration t_a, as a function of λ is shown in Fig. 6(a). Observing it, we verify that the mean of the total number of sticks increases as λ grows. As we known from Fig. 5 that the stick duration decreases as λ grows, we conclude that as λ grows, the system response presents on average a higher number of sticks, but these sticks have on average a lower duration. Given that, we may ask what happens with the total time of stick as λ grows. Computing the sum of the duration of all sticks, and dividing it by the duration t_a, we get the total time of stick in relation to the time interval analyzed. We call this random variable R.

The question is what happens with the mean of R when λ grows. Is it possible that, on average, a higher number of sticks with lower duration give us a higher total time of stick than a lower number of sticks with a higher duration? To answer this question, we plotted the estimated mean of R as function of λ. The obtained graph is shown in Fig. 6(b). Observing it, we verify that the mean of total time of stick reaches a maximum value, $\hat{\mu}_R^\star$, which is almost 80% and occurs at $\lambda = 3.8$ 1/s. The conclusion is that for $\lambda \in [0.5, 3.8]$ 1/s, the increase of the number of sticks causes the increase of the total time of stick, even though the sticks have a lower duration. The larger number of sticks compensates its shorter duration. However, when λ exceeds 3.8 1/s, the increase of the number of sticks compensates no more the reduction of its duration, so $\hat{\mu}_R^\star$ decreases.



(a) (b)

Figure 6: (a) Estimated mean of the number of sticks, and 95 % confidence interval as a function of λ. (b) Estimated mean of the total stick duration, $\hat{\mu}_R$, and 95 % confidence interval as a function of λ.

7 Influence of the friction coefficient on the statistical model

To quantify the influence of μ on the statistical model of the stick-slip process, we analyzed the graph of the estimated mean of the total time of stick $\hat{\mu}_R$ as a function of λ, Fig. 7. Observing it, we verify that the maximum of the estimated mean of total time of stick, $\hat{\mu}_R^\star$, grows as μ increases. For $\mu = 0.5$, the maximum is 15.0%, and for $\mu = 7.0$, the maximum is 81.89%. Besides, we verify also that the maximum is always reached for λ in the short interval [3.5, 3.8] 1/s. From these results, we conclude that the friction coefficient has a lot of influence on $\hat{\mu}_R$. However, very little influence on the position of the maximum.

8 Conclusions

The obtained results showed that the normalized histograms of the stick duration are sensitive to variations on λ and μ. As λ grows, the estimated mean of the stick duration decreases. Besides of this, the system response presents on average a higher number of

DOI: 10.5540/03.2018.006.01.0430

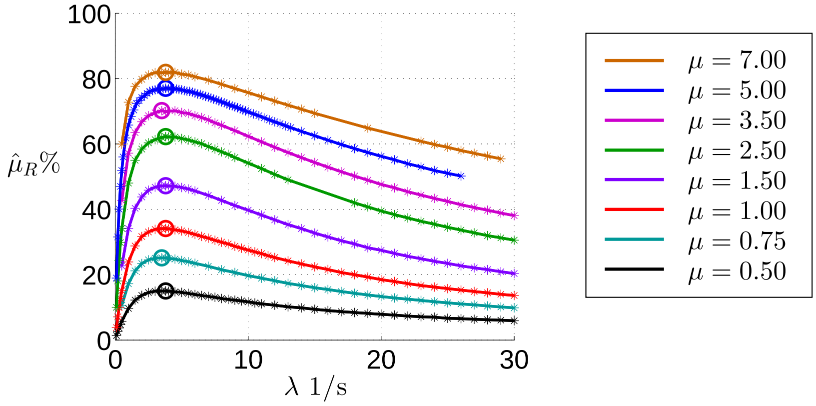


Figure 7: Estimated mean of the total stick duration $\hat{\mu}_R$ as a function of λ for different values of μ. The $\hat{\mu}_R^\star$ is highlighted for each μ with markers.

sticks, however these sticks have on average a lower duration. The relative total time of stick, R, showed that for λ lower than approximately 3.8 1/s, the increase of the number of sticks causes the increase of the total time of stick, even though the sticks have a lower duration, in a way that the large number of sticks compensates its shorter duration. However, when λ exceeds approximately 3.8 1/s, the increase of the number of sticks does not compensate the reduction of its duration anymore, so the total time of stick decreases. From the analysis of the influence of μ, we concluded that maximum of the estimated mean of total time of stick, $\hat{\mu}_R^\star$, grows as μ increases. However, the value of μ does not change the behavior of R in relation to the position of its maximum.

Acknowledgments

The authors acknowledge the support given by FAPERJ, CNPq and CAPES.

References

[1] M. Bengisu and A. Akay. Stick-slip oscillations: dynamics of friction and surface roughness, *Journal of the Acoustical Society of America*, 105(1):194–205, 1999.

[2] R. Lima and R. Sampaio. Stick-mode duration of a dry-friction oscillator with an uncertain model, *Journal of Sound and Vibration*, 353:259–271, 2015.

[3] R. Lima and R. Sampaio. Construction of a statistical model for the dynamics of a base-driven stick-slip oscillator, *Mechanical Systems and Signal Processing*, 91:157–166, 2017.

[4] R. Lima and R. Sampaio. Parametric analysis of the statistical model of the stick-slip process, *Journal of Sound and Vibration*, 397:141–151, 2017.

