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A Mathematical Model for Behavioral Epidemiology: A Numerical Approach

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Abstract. In this work, we propose a new basic mathematical model for Behavioral Epidemiology. It is based in a SIRS epidemiological model and in the Replicator Equations. We use numerical simulations to explore some distinguished characteristics of the model.

Palavras-chave. Behavioral Epidemiology, SIRS model, Replicator Equations.

1 Introduction

The key issue in behavioral epidemiology is to understand how populational behavior is related with the development of diseases. In particular, the relationship between behavioral changes and the dynamics of infectious diseases is a fundamental question.

Mathematical modeling in epidemiology, is already a well established practice [1] and though mathematical modeling in behavioral epidemiology is not a new field [2], has not been explored in the same way. In this work, we propose a basic model to consider epidemiological and behavioral factors simultaneously and explore numerically some of its distinguished characteristics.

In the following sections we rapidly review some basic epidemiological model as well as one of the most used models in behavioral dynamics: the Replicator Equations. Later, based on those ideas we propose a basic behavioral epidemiological model and consider numerical simulations to explore some of its properties.

2 SIRS Epidemiological Model

One of the most basic epidemiological model for infectious diseases is the SIR model. There are several models that can be found on the literature under this name, but all of them are based on the idea of dividing a population into three classes: susceptible, infected and recovered [1]. Some models describe how the fraction of a population in each of these classes changes with time. Alternatively the model can consider the number of individuals in each class. Classical SIR model consider that the disease is not lethal and that total population remains constant. Over a sufficiently short time scale, this last assumption is

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reasonable, but to capture the dynamics over the longer term this assumptions must be overruled. Several refinements of the SIR model are possibles but in this work we consider a SIRS model with death and birth rates such that if S, I, R refer to the population in each compartment, and $N = S + I + R$ is the total population, these variables change according to the following system of differential equations:

$$\begin{aligned} \frac{dS}{dt} &= \mu_B N - \frac{\beta\, S\, I}{N} - \mu_D\, S + \kappa\, R \\ \frac{dI}{dt} &= \frac{\beta\, S\, I}{N} - \gamma\, I - \mu_D\, I \\ \frac{dR}{dt} &= \gamma\, I - \mu_D\, R - \kappa\, R. \end{aligned} \tag{1}$$

Here, γ represents the recovery rate, the force of infection is given by $\beta \frac{I}{N}$ and represents the rate of susceptible individuals that became infected. The parameter β can be interpreted as the contact rate of infection. The parameters μ_B, μ_D represents birth and death rate respectively and most models consider $\mu_B = \mu_D$. The parameter κ represents the rate of going from recovered to susceptible status. In general, models based on the SIR model predict asymptotic convergence to an equilibrium state. The SIRS model also has these characteristic but show some oscillatory behavior at earlier times. Some typical trajectories for SIRS models are presented in the Figure 1.

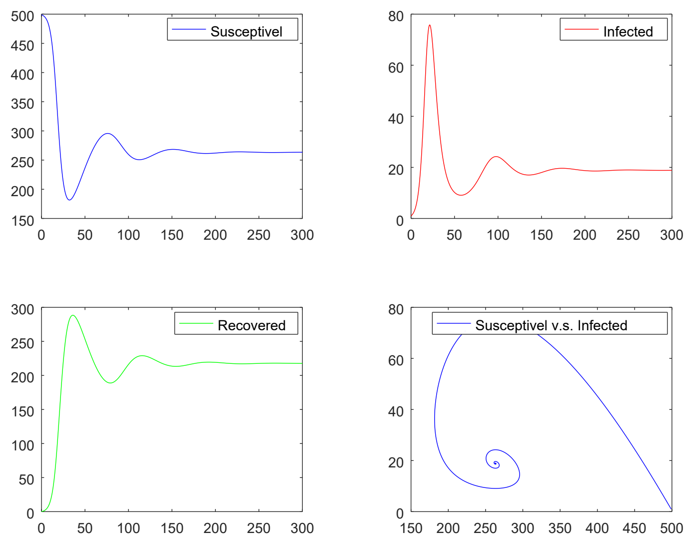

Figura 1: Trajectories for SIRS Model with $\mu_B = \mu_D = 0.016, \gamma = 0.3, \beta = 0.6, \kappa = 0.01, S(0) = 499, I(0) = 1, R(0) = 0$.

3 Replicator Equations

The continuous replicator equations are a set of differential equations originally used in evolutionary game theory to represent the evolution of mutually exclusive strategies or behaviors across a population of non-necessarily rational individuals [3]. It is based on the idea that the success of a strategy in the population, relies on how the strategy payoff compares with others payoff, given a current distribution of strategies. In this sense, replicator equations try to mimic the evolutionary process of survival of the fittest. Assume x_i is the proportion of the population following behavior i and $f_i(x)$ is the payoff of following strategy i when the behavioral distribution is $x = (x_1, \ldots, x_n)'$. The replicator equations are defined by the following system of differential equations:

$$\frac{dx_i}{dt} = x_i[f_i(x) - \overline{f}(x)] \tag{2}$$

where $\overline{f}(x)$ is the average population payoff (fitness) defined by $\overline{f}(x) = \sum_{i=1}^{n} x_i f_i(x)$.

System 2 means that the *per capita* rate of growth for followers of strategy i, depends on the difference between the current payoff f_i and the current average payoff $\overline{f}$. In some situations, the payoff is assumed to depend linearly upon the population distribution, which allows the replicator equations to be written in the form:

$$\frac{dx_i}{dt} = x_i \left((Ax)_i - x^T Ax \right), \tag{3}$$

where the payoff matrix A holds all the payoffs information for the population. The (expected) payoff $f_i(x)$ of strategy i, is given by $(Ax)_i$ and the average population payoff $\overline{f}(x)$ is given by $x^T Ax$ [3].

As an example of the replicator equation applied to an evolutionary game, we present the evolutionary version of the well-known game called Rock-Scissor-Paper. In this game, the members of the population are continuously playing in one-to-one random encounters between anonymous players. At every time a player can choose some of the strategies: *(R) Rock*, *(S) Scissors*, or *(P) Paper*. The payoffs for the one-to-one encounters are given in the matrix:

$$A = \begin{array}{c} \\ Rock \\ Scissor \\ Paper \end{array} \begin{array}{c} \begin{array}{ccc} Rock & Scissor & Paper \end{array} \\ \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \end{array}. \tag{4}$$

A player using strategy i against a player using strategy j get a payoff given by a_{ij}, so for example, an encounter between a *Paper* player and *Rock* player, results in a payoff of 1 unity for the *Paper* player and -1 for the *Rock* player. Replicator equations pretend to describe how the distribution of strategies are evolving. In a population with a initial majority of Rock players, the best strategy would be to play Paper, so the population tend to evolve to this behavior. With a majority of Paper players, this is strategy is no longer the best one, but instead Scissors players should start to increase. When the replicator equations (3) are applied to this game, the strategies behave in cyclical trajectories and have a periodic behavior, so the states x_1, x_2 and x_3 are the parametric equations of a

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closed trajectory contained on the set $\{x = (x_1, x_2, x_2) \in \mathbb{R}^3 : x_i \geq 0,\ x_1 + x_2 + x_3 = 1\}$. Typical trajectories are pictured in the Figure 2.

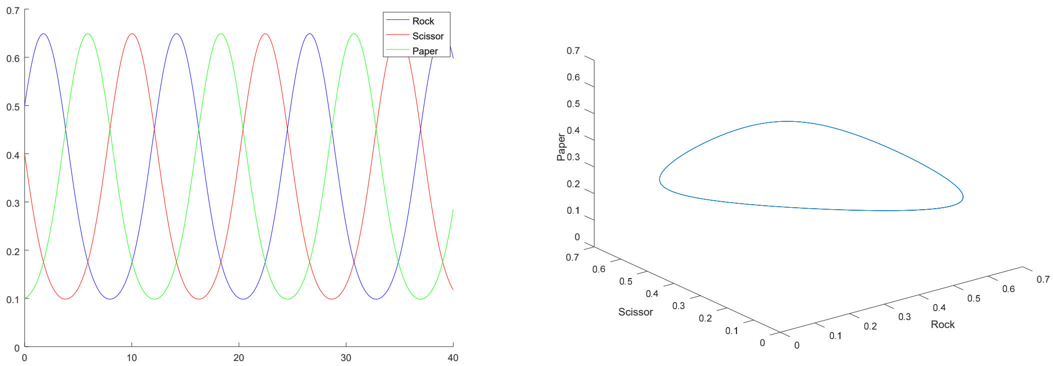

Figura 2: Trajectories for the Rock-Scissor-Paper Game with Replicator Equation with $x_1(0) = 0.5, x_2(0) = 0.4, x_3(0) = 0.1$.

4 A Basic Model for Behavioral Epidemiology

Based on the SIRS model and on the Replicator Equations we can consider a basic model for behavioral epidemiology. Besides the epidemiological variables $X = (S, I, R)$, we consider three mutually exclusive behaviors B_1, B_2 and B_3. We are going to assume initially that the behaviors B_1, B_2 and B_3 are related respectively to avoiding infection, increasing recovery after infection, and increasing immunization after recovery. We are assuming also that every member of the population is following one and only one of the possibles strategies. If x_i refer to the proportion of population following the behavior B_i, we propose the following model:

$$\begin{aligned}
\frac{dS}{dt} &= \mu_B N - \frac{(1-x_1)\beta\, S\, I}{N} - \mu_D\, S + (1-x_3)\kappa\, R \\
\frac{dI}{dt} &= \frac{(1-x_1)\beta\, S\, I}{N} - x_2\,\gamma\, I - \mu_D\, I \\
\frac{dR}{dt} &= x_2\,\gamma\, I - \mu_D\, R - (1-x_3)\kappa\, R. \\
\frac{dx_i}{dt} &= x_i[f_i(x, X) - \overline{f}(x, X)] \quad n = 1, 2, 3.
\end{aligned} \tag{5}$$

This model can be seen first as an SIRS model for the epidemiological variables, where the transition rates are influenced by behavioral variables x_i. So for example, the infection parameter β is being multiplied by $(1 - x_1)$ in the first and second equation, which means that if 100 percent of the population follows the behavior B_1 there would be not new infected. In the same way, if all population follows behavior B_2, the recovery rate is increased, as is implied by the term $x_2\,\gamma\,I$, in the second and third equation. If all population follows behavior B_3, then there will be full immunization described by the term $(1-x_3)\kappa\,R$ in the first and third equation. Finally, the last equation set that the replicator equations is used for the behavioral variables x_i, and also that the epidemiological variables $X = (S, I, R)$ can influence the behavioral dynamics via payoff functions $f_i(x, X)$.

In order to illustrate the model (5), we consider first a simple case, when the behavioral payoffs are independent of the epidemiological variables and the behaviors follows the Rock-Scissor-Paper Dynamic. This may be the case, if behaviors across the population are following some periodical trends, but in general this may not be a realistic assumption. Some trajectories are pictured in the Figure 3.

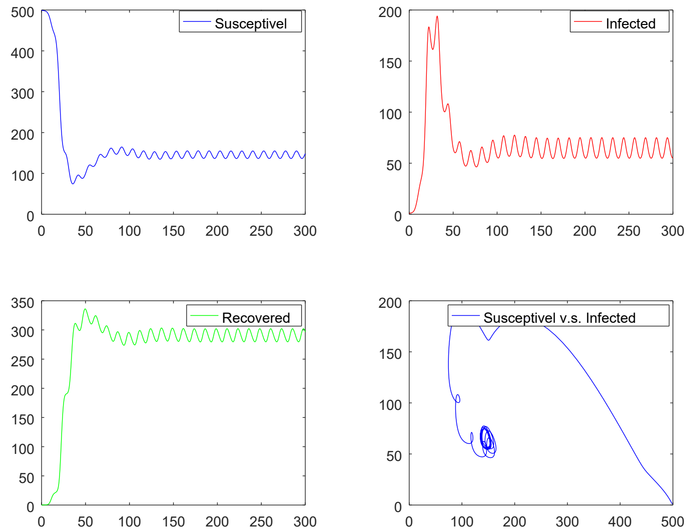

Figura 3: SIRS+Replicator Model with Rock-Scissor-Paper Behavior and $\mu_B = \mu_D = 0.016, \gamma = 0.3, \beta = 0.6, \kappa = 0.01, S(0) = 499, I(0) = 1, R(0) = 0,\ x_1(0) = 0.5, x_2(0) = 0.4, x_3(0) = 0.1$.

As we see, the trajectories depicted by the model, has some resemblance to the ones presented in Figure 1 but there are some differences. The model replicate the initial steady increase in the infected population but predicts an increase in the recovered population as a consequence of the behavioral considerations. The model predicts also a continuous oscillatory behavior.

We can consider now some different dynamics. For example, assume that instead of static matrix A as (6), the payoff of behaviors are given by a dynamic matrix A_X that depends on the epidemiological variables $X = (S, I, R)$ as the following:

$$A_X = \begin{array}{c} \\ \textit{Behavior 1} \\ \textit{Behavior 2} \\ \textit{Behavior 3} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{Behavior 1} & \textit{Behavior 2} & \textit{Behavior 3} \end{array} \\ \begin{pmatrix} 0 & \frac{S-I}{N} & \frac{S-R}{N} \\ \frac{I-S}{N} & 0 & \frac{I-R}{N} \\ \frac{R-S}{N} & \frac{R-I}{N} & 0 \end{pmatrix} \end{array}. \tag{6}$$

The entrance of payoff matrix A_X can be interpreted as the advantage obtained by a behavior when compared with the other ones, in the epidemiological situation $X = (S, I, R)$. Note that the Behavior 1, associated with remaining in the susceptible group (avoiding infection), increase its advantage if the number of susceptibles increase. In the same way, the Behavior 2, related to leave the infected group, increase its advantage if the infected population increase. The Behavior 3, related to remains in the recovery group, increase its advantage if population in recovery increase. The matrix A_X can therefore be interpreted as incentives for doing what seems the right thing in the right situation, a *perfectly right dynamics* in some sense. Figure 4 presents the trajectories for this model considering the same parameters and initial conditions that in the example pictured in Figure 3.

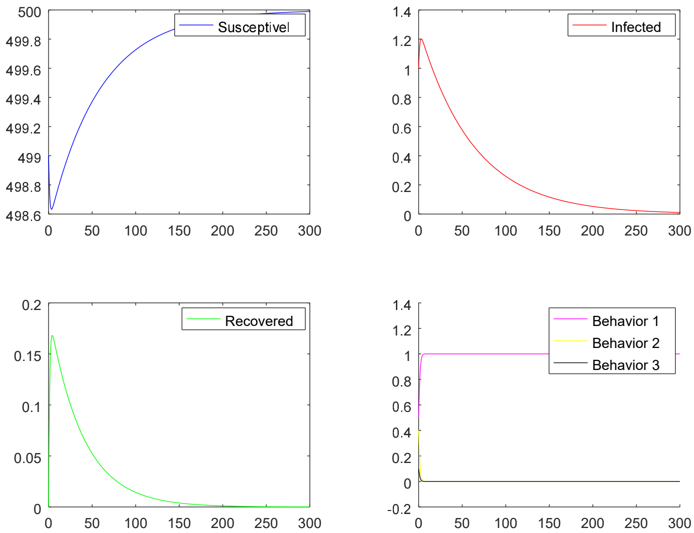

Figura 4: SIRS+Replicator Model with Perfectly Right Dynamics and $\mu_B = \mu_D = 0.016, \gamma = 0.3, \beta = 0.6, \kappa = 0.01, S(0) = 499, I(0) = 1, R(0) = 0,\ x_1(0) = 0.5, x_2(0) = 0.4, x_3(0) = 0.1$.

Figure 4 may seem surprisingly straightforward compared with Figure 3. In both situations we are considering the same dynamics for the epidemiological variables so the main difference is the use of the matrix A_X. The trajectories depicted in Figure 4 it would represent an utopical situation in the long-term (infection was contained and the Behavior 1, that avoid new infections, is spread across population). This of course is not a realistic expectation in many situations. The most important insight we can obtain from this numerical approach, is that **a proper policy of dynamical incentives may improve diseases control**. But, what would happen if no one is ever following the preventive behavior B_1? Can behaviors B_2 and B_3 be enough for controlling the disease? The answer may be negative, as is suggested by the results presented in Figure 5, corresponding to the *perfectly right dynamics* with initial conditions $x_1(0) = 0.0, x_2(0) = 0.5, x_3(0) = 0.5$

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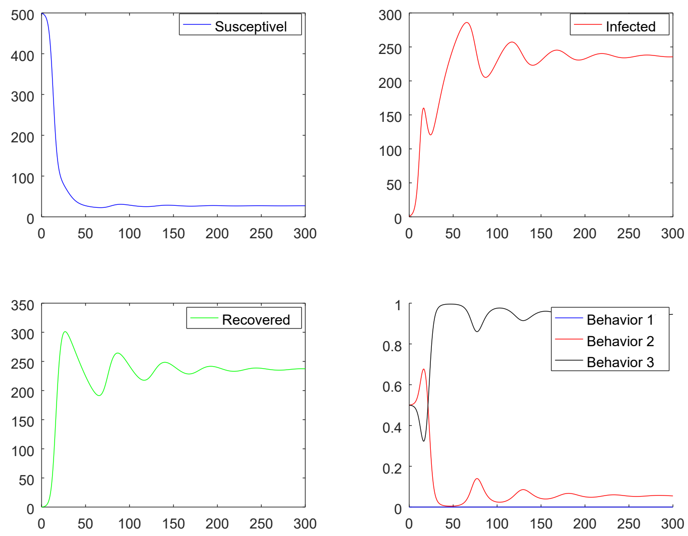

Figura 5: SIRS+Replicator Model with Perfectly Right Dynamics and $\mu_B = \mu_D = 0.016, \gamma = 0.3, \beta = 0.6, \kappa = 0.01, S(0) = 499, I(0) = 1, R(0) = 0,\ x_1(0) = 0.0, x_2(0) = 0.5, x_3(0) = 0.5$.

The previous questions and other issues raised in the present work, will be deeply explored numerically and analytically in future works. The mathematical framework proposed in this work can be improved and adapted to several situations. The assumption that all behaviors were *well-intended* to reduce the disease impact, may be substituted for a more realistic situation, where some of the behaviors may actually increase the risk of infection or decrease the rate of recovery. Also, non-matrix payoff should be considered to obtain most realistic models.

Acknowledgements

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Referências

[1] M. J. Keeling, P. Rohani. *Modeling Infectious Diseases in Humans and Animals*. Princeton University Press, 2011.

[2] P. Manfredi and A. d'Onofrio (Eds.). *Modeling the Interplay Between Human Behavior and the Spread of Infectious Diseases*; Sipriger, 2013.

[3] J. W. Weibull. *Evolutionary Game Theory*. MIT Press, 1997.

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