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A Mathematical Model and Heuristic Approach for the Production Planning in the Glass Container Industry

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Abstract. This work presents a mathematical model for production planning problem in the glass containers industry. The manufacturing process in this type of industrial plant has two phases. In the first phase, the glass melting in a furnace is performed and in the second stage the production of containers occurs in molding machines. The appropriate configurations for each machine connected in a furnace is a decision to be taken, which depends on the demand forecast for glass container within a time horizon. The model proposed will describe objectives and constraints that allow defining the best configuration for the whole plant. A heuristic combining *Fix-and-Optimize* and *Relax-and-Fix* approaches is also applied over the same instances. The results indicate that the proposed model as well as the heuristics are able to provide good quality solutions for this problem.

keywords. Glass Container Industry, Mathematical Modeling, Production Planning, Heuristics, Relax-and-Fix, Fix-and-Optimize.

1 Introduction

The problem studied on this paper deals with how to determine a better configuration for an industrial plant that produces glass containers. The lot-sizing decisions must satisfy constraints related with machines and furnace capacities. Currently, production planning can reflect the possible specificity of industrial processes and allow greater flexibility for production operations [10].

The main contribution of this work is to propose a mathematical formulation, based on mixed integer programming model, capable of describing the production situation studied on this paper. A set of instances is generated from data provided by a glass container

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industry located in Portugal. The heuristics is based on mathematical programming and combines the Relax-and-Fix (R&F) along with Fix-and-Optimize (F&O) approaches.

2 Related Work

The glass container problem considered in this paper includes lot-sizing decisions. There are several reviews in the literature about lot sizing and production planning as [6] and [12]. A review about models for lot sizing problems is presented in [10], while another review considering meta-heuristics applied to lot sizing problems can be found at [11]. In the context of lot-sizing problem in the glass container industry, the authors in [1] treat a system with multiple production facilities where a mathematical model is presented. In [2] and [15], it is described a mathematical formulation and meta-heuristic methods for the short-term production planning in a single-facility with only one furnace.

The present paper applies a heuristics based on mathematical programming that combines the construction heuristics *relax-and-fix* (R&F) with the improvement heuristics *fix-and-optimize* (F&O) . A review on mathematical programming based heuristics is presented in [4]. The construction method R&F solves mixed-integer programming (MIP) sub models, optimizing some integer variables and keeping others relaxed [17]. The authors in [3] applies R&F heuristics to solve a lot sizing problem in the foundry industry. A similar approach is applied by [16] and [5] for a lot sizing problem in the animal feed industry. In [7] and [8], R&F is also used to solve a production problem in the beverage industry. The improvement F&O heuristics starts from an initial solution, wheres some of its variables are re-optimized while others are fixed [17]. In [14], F&O was combined with an genetic algorithm to solve also a multi-stage lot sizing problem.

3 Mathematical model

This section presents the mathematical model, where it is assumed only one furnace that can be connected to several machines with different configurations. The parameters and decision variables are shown next.

Parameters:

- m: Machines available ($m = 1, ..., M$).
- i: Products to be manufactured ($i = 1, ..., I$).
- a: Annual Time horizon ($a = 1, ..., A$).
- NS_m: Number of sections by machine m.
- TG_m: Type of gob by machine m.
- AC_{im}: 1 if product i is accepted in the machine m.
- C_m: Cost to install machine m. ($)

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- D_{ia}: Demand expected of product i in period a. (ton)
- W_i: Weight of product i. (ton)
- R_i: Efficiency of the cavity for product i. (bottles/min)
- $\overline{M}$: Maximum machines supported by the new furnace.
- CF: Cost to install fuse capacity on furnace. ($/ton)
- η_m: Efficiency of machine m. (%)

Variables:

- KF: Melting capacity required for the furnace. (ton)
- Q_{ima}: Lot size of product i on machine m in the period a. (ton)
- F_{ima}: Time spent on period a in which machine m was dedicated to produce product i. (years)
- $\overline{Y}_m$: 1 if the machine m is installed, 0 otherwise.

Formulation:

$$Min \quad f(KF, \overline{Y}_1, ..., \overline{Y}_M) = CF * KF + \sum_{m=1}^{M} C_m.\overline{Y}_m \tag{1}$$

Subject to:

$$\sum_{m=1}^{M} \overline{Y}_m \leq \overline{M} \tag{2}$$

$$F_{ima} \leq \overline{Y}_m \qquad \forall(i,m,a) \tag{3}$$

$$F_{ima} \leq AC_{im} \qquad \forall(i,m,a) \tag{4}$$

$$\sum_{i} F_{ima} = \overline{Y}_m \qquad \forall(m,a) \tag{5}$$

$$Q_{ima} = F_{ima}.(R_i.W_i.NS_m.TG_m.\eta_m) \qquad \forall(i,m,a) \tag{6}$$

$$\sum_{\tau=1}^{a} \sum_{m} Q_{im\tau} \geq \sum_{\tau=1}^{a} D_{i\tau} \qquad \forall(i,a) \tag{7}$$

$$\sum_{i} \sum_{m} Q_{ima} \leq KF \qquad \forall(i,a) \tag{8}$$

$$KF, Q_{ima}, F_{ima} \geq 0 \tag{9}$$

$$\overline{Y}_m \in \{0,1\} \tag{10}$$

The objective function (1) minimizes costs related to increase the melting capacity of the furnace along with the cost of machines. Constraints (2) represent the limit of machines that can be installed in the furnace. Constraints (3) allow the production of product i only if machine m is installed, while constraints (4) allow the production only if the product i is accepted by machine m. The setup time between products for each period a and machine m is described by constraints (5). The lot-size of product i in machine m in period a is defined in constraints (6), while constraints (7) ensure the demand satisfaction. Constraints (8) define that the furnace and machine capacities must meet the demand. The variable domains are given by constraints (9) and (10).

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4 Methods

Algorithm 1 describes the R&F developed, where a partition ($\hat{m}$) defines the set of binary variable to be optimized while the others remain relaxed. At the end of each iteration, the binary variables have their values fixed and another partition is defined.

Algorithm 1: *Relax–and–Fix*

begin
    Solve model with all $\overline{Y}_m$ variables relaxed;
    $\hat{m} \leftarrow 0$
    for $\hat{m} \leq M$ **do**
        Variables $\overline{Y}_m$ become binary ;
        Solve model;
        if *there is solution found* **then**
            Fix $\overline{Y}_m$ values;
            $\hat{m} \leftarrow \hat{m} + 1$
        end
        else
            Infeasible
        end
    end
end

Algorithm 2 describes the F&O, where the final solution of R&F is used as initial solution. Of course, if R&F did not find a feasible solution, F&O will be also infeasible once it is a improvement heuristic and needs an initial solution. A partition ($\hat{m}$) defines the set of binary variable to be optimized while the others remain relaxed. At the end of each iteration, the binary variables have their values fixed and another partition is defined.

Algorithm 2: *Fix–and–Optimize*

Final R&F solution
$currentSolValue \leftarrow rfSolValue$
begin
    $\hat{m} \leftarrow 0$
    for $\hat{m} \leq M$ **do**
        Set free to optimize the binary variables $\overline{Y}_m$
        Keep fixed the values of other binary variables
        Solve model
        if $solValue \leq currentSolValue$ **then**
            $currentSolValue \leftarrow solValue$
            Fix $\overline{Y}_m$ values;
        end
        else
            Fix previous non-optimized $\overline{Y}_m$ values
        end
        $\hat{m} \leftarrow \hat{m} + 1$
    end
end

5 Computational Results

The instances to be evaluated were created by a random generator from data provided by a glass container industry located in Portugal. Two groups of instances were elaborated: Factory-Machine (FM) and Horizon-Time (HT). A total of five subgroups of instances were created for each case, where each subgroup contains 10 instances.

The mixed integer model proposed was coded in C/C++ using the interface OPL of package IBM ILOG CPLEX 12.6. The interface OPL and solver CPLEX were also used to develop the heuristics R&F with F&O. The exact solution from model formulation and the solutions returned by R&F with F&O were tested in a total of 100 instances, where 50 instances come from FM and 50 from HT. The time limit was one hour to run each method. All tests were performed on a computer with an Intel(R) Core(TM) i7, 2,67 GHz e 18 GB RAM, and operating system Linux (version 3.2.0-4-amd64 / Debian 4.6.3-14). The resolution of instances from the proposed model, using the solver CPLEX, allowed reaching optimal solutions for some instances as can be observed in Figure 1.

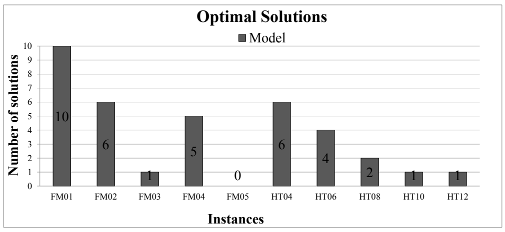

Figure 1: Number of optimal solutions using the proposed model.

In Figure 1, it is observed that solver CPLEX finds 10 optimal solutions in the scenario with 100 machines (FM01). In other scenarios, it was possible to find at maximum 6 optimal solutions. In the HT group, a total of 6 and 4 optimal solutions are obtained for the two simpler scenarios (HT04 and HT06), while we have two or one optimal solution returned within one hour for the other cases. The number of variables and constraints increases for both cases when parameters M and T are increasing. This directly impacts over the exact method performance.

$$Gap(\%) = 100 * \frac{CPLEX^{UB} - CPLEX^{LB}}{CPLEX^{UB}} \tag{11}$$

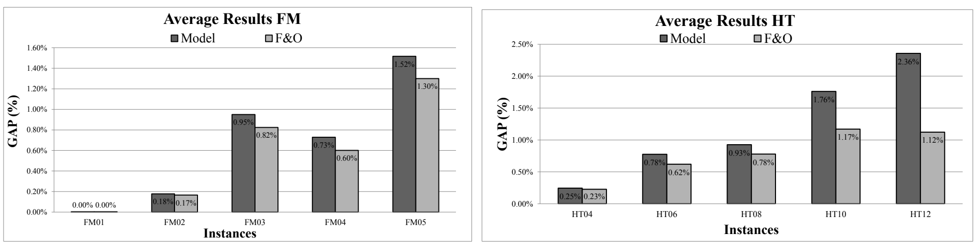

Figure 2: Average results obtained through GAP.

Figure 2 compares the average results obtained through the GAP defined in equation (11). This value is obtained from the difference between the Upper Bound (UB) and Lower Bound (LB), divided by the Upper Bound (LB). For instances of the FM group, the methods were the same for FM01 subgroup, while the R&F with F&O heuristics obtained better solutions on average than solver CPLEX for the other subgroups. In the HT instances, the proposed heuristic got better solutions for all subgroups.

Figure 3 presents the number of better solutions returned by some of the methods and the amount of draws. A draw occurs when $|solMethod1 - solMethod2| < 0.01$.

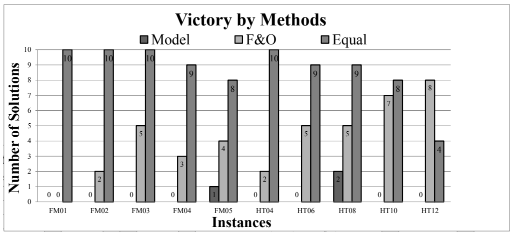

Figure 3: Number of victory and equal between methods.

R&F with F&O heuristics wins more than CPLEX for FM group, but several draws also happend. For HT group, there is less number of draws and the propsed heuristic confirms its superiority in terms of number of wins.

6 Conclusion

This work introduced a mathematical model where decisions about how to define a plant configuration can be done along with lot size decisions. The mathematical model was able to solve optimally only subgroup FM01. The proposed heuristics is promising once its average GAPs are better for 9 out of 10 sub-groups. There are several draws among the final solutions returned by using the mathematical formulation introduced and the proposed heuristics. However, R&F with F&O heuristics returned the best solutions when there are no draws.

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