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Numerical Simulation of Fins in a High Temperature Context

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Abstract. The present work shows the influence of the mutual heat transfer on the effectiveness of finned surfaces. Numerical simulations are carried out through a sequence of linear problems, possessing an equivalent minimum principle, that has as its limit the solution of the original problem. The presented tools allow the employment of realistic hypotheses. The problems are simulated with the aid of a finite difference approximation. This work accounts for the the steady state heat transfer process in rigid fins which experiences convective and radiative heat exchange. Some typical results are shown in order to illustrate the methodology. Results indicate that mutual radiation can significantly impact the actual heat transfer response of a fin.

Keywords. Nonlinear Heat Transfer, Numerical Simulation, Sequence of Linear Problems, Finite Difference Method.

1 Introduction

The main techniques of heat transfer enhancement are usually those that enhance existing heat transfer elements. A growing demand for engineering projects is related to energy transitions that require a rapid flow of heat. Modern times require high performance heat transfer components with progressive weight reduction, volume and cost. Heat transfer on extended surfaces comes to be the study of these components of high performance heat transfer with respect to the most varied parameters and their respective behaviors in the thermal mapping.

This work proposes a study about the interaction between fins and primary surface, mutual radiation between adjacent fins and the combined effect of mutual radiation and environmental radiation. Mutual radiation has a fundamental role in projects related to photovoltaic panels, aircraft, satellites, thermoelectric generators, thermal treatments of components and in the most different applications industries involving high temperatures. Recent studies have shown the relevance of such considerations in the most diverse engineering applications.

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Bjork et al. (2014) [2] investigated different mechanisms of heat loss in a thermoelectric generator. The model works with non-linear boundary conditions, including mutual radiation between adjacent surfaces. Alifanov et al. (2016) [1] presented a methodological approach for the thermal mapping of a small spacecraft. The mathematical model of heat transfer considers the external radiant flux field and the influence of mutual radiation between spacecraft surfaces. Cheema et al. (2016) [3] investigated numerically the heat transfer in tubular furnaces considering mutual radiation properties among their constituents. Tubular furnaces are used in the annealing of crystalline structures of metals at high temperatures and the determination of the temperature distribution inside a tubular furnace is an important coefficient for efficiency in the heat treatment.

The present work numerically investigates the performance of coupled heat transfer in longitudinal fins arrays considering mutual radiation between radiant elements. Conduction-radiation heat transfer process is an inherently non-linear phenomenon in which the coupling in the contour of the body is mathematically represented by a non-linear relationship between the absolute temperature and its outside normal derivative, in which the unknown is the temperature distribution . The solution to the problem is given by the limit of a sequence whose elements are obtained from the minimization of a quadratic functional.

2 Numerical Formulation

Enhancement heat transfer techniques have been the subject of multiple studies for several geometries. Experimental procedures are more costly and may present different results from standards, due to the difficulty of working with low temperature differences and/or high coefficients of heat transfer. Therefore, numerical simulation techniques are required to obtain effective and realistic results.

2.1 Mathematical model

Krauss (2002) [7] shows that steady-state analyzes become realistic in most problems concerning extended surface applications, some specific cases such as high-speed aircraft applications and automatic control devices require care with transient term.

Therefore, the steady-state heat equation without internal heat generation with constant thermal conductivity and non-linear boundary conditions for single fin is

$$\frac{\partial}{\partial x}\left(\frac{\partial \overline{T}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial \overline{T}}{\partial y}\right)-\frac{2}{\delta k}\left[h\left(\overline{T}-T_{\infty}\right)+\epsilon\sigma|\overline{T}|^{3}.\overline{T}\right]=0 \quad \text{in} \quad \Omega_1 \tag{1}$$

in a configuration described by $0 < x < L$, $0 < y < b$ e $0 < z < \delta$.

For two fins in steady state, as shown in Fig. 1, without internal heat generation with constant thermal conductivity and nonlinear boundary conditions, taking into account mutuality in the emission of radiation between adjacent fins, combined effect of mutual and environmental radiation, in addition to the interaction between the fins and their

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respective primary surfaces, the heat equation becomes

$$\frac{\partial}{\partial x}\left(\frac{\partial \overline{T}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial \overline{T}}{\partial y}\right)-\frac{2}{\delta k}\left[h\left(\overline{T}-T_{\infty}\right)+\epsilon\sigma|\overline{T}|^{3}.\overline{T}-\frac{1}{2}\int \sigma|\overline{T}|^{3}.\overline{T}\kappa_{12}dS\right]=0 \tag{2}$$

for dimension L much greater than b, we have for $0<y<b$

$$\frac{d^2\overline{T}}{dy^2}-\frac{1}{\delta k}\left[2h\left(\overline{T}-T_{\infty}\right)+2\sigma\left|\overline{T}\right|^{3}\overline{T}-\int_0^b \sigma\left|\overline{T}(\xi)\right|^{3}\overline{T}(\xi)\left(\frac{d^2}{2\left((y-\xi)^2+d^2\right)^{3/2}}\right)d\xi\right]=0 \tag{3}$$

Eq. 4 is a portion referring to the emission of radiation between fins, as shown

$$\int_0^b \sigma\left|\overline{T}(\xi)\right|^{3}\overline{T}(\xi)\left(\frac{d^2}{2\left((y-\xi)^2+d^2\right)^{3/2}}\right)d\xi \tag{4}$$

$k(T)$ is the thermal conductivity of the regions Ω_i, κ_{12} the form factor, ξ the term from the kirchhoff transform for problems with non constant thermal conductivity and d the distance between the fins. The boundary conditions (b.c.) are homogenous of Neumann in Γ_1, Γ_3 e Γ_4 and Dirichlet b.c. in Γ_2. Equations 1, 2, 3 e 4 employ the term $|T|^3.T$ in place of term T^4 to guarantee operator coercivity of infinite dimension, thus preserving the physical structure of the phenomenon of heat transfer.

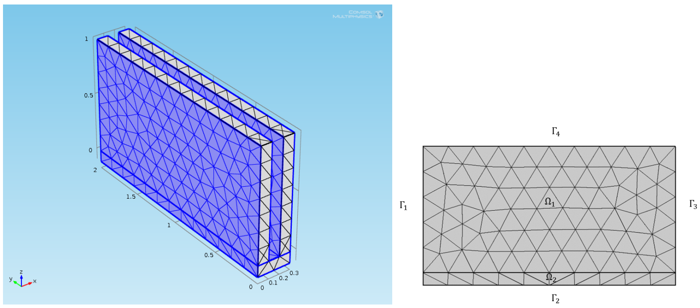

Figura 1: - Dual fins and boundary conditions.

2.2 Sequence of problems

In order to ensure the existence of a minimum, solution for Eq. 1 and Eq. 2, is necessary and sufficient to show that I, is continuous, convex and coercive functional.

The solution of problem in Eq. 1 may be reached as the limit of a sequence whose elements ate obtained form the minimization of a quadratic functional.

$$I[\nu]=\frac{1}{2}\int_0^b\int_0^L\left[\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]dxdy+\int_0^b\int_0^L\left[\frac{h}{\delta k}\left(\nu-T_{\infty}\right)^2+\frac{2\varepsilon\sigma}{5\delta k}|v|^5\right]dxdy \tag{5}$$

The first variation of Eq. 5 describes the variational formulation of the physical problem, which must be represented mathematically by minimizing the coercive functional. The existence of the minimum principle provides a simple and precise tool for numerical simulation of the phenomenon of heat transfer.

In other words:

$$\omega = \lim_{i \to \infty} \Phi_i \tag{6}$$

in which the elements of the sequence $[\Phi_0, \Phi_1, \Phi_2, ..., \Phi_i]$ are obtained by

$$div\,(grad\Phi_{i+1}) = \alpha\Phi_{i+1} - \beta_i \quad \text{in} \quad \Omega_1 \tag{7}$$

and

$$-\,(grad\Phi_{i+1})\,\boldsymbol{n} = 0 \quad \text{on} \quad \partial\Omega_1 \tag{8}$$

the β auxiliar term:

$$\beta_i = \alpha\Phi_{i-1} - \left(\sigma\,|\Phi_{i-1}|^3\,\Phi_{i-1} - h\,(\Phi_{i-1} - T_\infty)\right) \quad \text{for} \quad i = 0, 1, 2, ... \tag{9}$$

where α is sufficiently large positive constant and $\Phi_0 \equiv 0$. This constant is evaluated from an priori estimate for the upper bound of the solution and ensures a bounded and nondecreasing sequence $[\Phi_0, \Phi_1, \Phi_2, ..., \Phi_i]$.

The problem proposed in this work originally has Robin boundary conditions, but the proposed methodology simplifies the convergence solution mode, imposing Neumann boundary condition without physical sense, but being an efficient mathematical tool.

In other words, the minimum of I_{i+1} is reached for the field $v = \Phi_{i+1}$ which satisfies in Eq. 1. It is to be noticed that Φ_i is a known function, when we look for the minimum of the functional. The tools employed for minimizing $I_{i+1}[v]$ are exactly the same ones employed for solving linear heat transfer problems. The constant α must satisfy the following relationship:

$$\alpha \geq \frac{d}{d\eta}|\eta|\left(h.e^\lambda + \sigma|e^\lambda|^3.e^\lambda\right) \quad \text{for} \quad 0 < \eta < \sup_{\partial\Omega} \omega \tag{10}$$

that is equivalent

$$\alpha \geq \frac{h}{\delta}, \quad \delta = e^\lambda + \sigma|e^\lambda|^3.e^\lambda \tag{11}$$

It is remarkable that any α satisfying Eq. 11 ensures convergence, but this is a sufficient condition. The convergence may be reached even for values of α which do not satisfy Eq. 11.

The sequences $[\Phi_0, \Phi_1, \Phi_2, ..., \Phi_i]$ and $[\beta_0, \beta_1, \beta_2, ..., \beta_i]$ are, for each i, non-decreasing, provides only the solution with physical sense. This fact allows us to conclude that the solution is the unique non-negative, since, from classical thermodynamics, the absolute temperature must be a non-negative value real field, we conclude that Eq. 1 and Eq. 7 are thermodynamically equivalent. The algorithm associated to the minimization of the functional I provide an efficient procedure for simulating the considered energy transfer phenomena. In addition, this procedure provides only the (desired) solution with thermodynamical sense.

3 Results and Discussion

Recent studies have reported numerical-experimental problems in multi-finned heatsinks with linear boundary conditions using multiphysics software [4] and [5]. Others obtained numerical solutions in FDM, FEM and FVM in the most diverse matrices neglecting radiation nonlinear boundary conditions [9] and [8]. Dogonchi (2016) [6] verified behavior of the steady state single rectangular fin with nonlinear boundary conditions and thermal conductivity varying linearly with temperature.

This work introduces the idea of the sequence of linear problems to find a solution of the steady-state problem in fins with non-linear b.c whose thermal conductivity is constant. Moreover, the mathematical analysis is based on the modifications of Murray-Gardner's hypotheses so that realistic problems are solved. Such procedures include the effects of: non-zero heatsink temperature, interaction between fin and primary surface, mutual radiation between adjacent fins, combined effect of mutual radiation with environmental radiation and the interaction of radiation with the associated structure.

3.1 Numerical convergence

Table 1 illustrates the numerical convergence process, presents Φ_1, Φ_2, Φ_3, Φ_4, Φ_5, Φ_6 and Φ_∞ obtained with three different values of α ($\alpha = \alpha_1 = 1.10^1 W/m^2K$, $\alpha = \alpha_2 = \alpha_1^2$ and $\alpha = \alpha_3 = \alpha_1^3$).

Tabela 1: - Numerical convergence verification

Φ_i	$\alpha = \alpha_1$	$\alpha = \alpha_2$	$\alpha = \alpha_3$
i=10	.5945	0.5941	0.5940
i=100	5.2373	5.2245	5.0802
i=1000	10.0053	10.0052	10.0021
i=1200	10.0081	10.0076	10.0059
i=1387	10.0081	10.0081	10.0079
$i \to \infty$	10.0081	10.0081	10.0081

3.2 Single and dual fins with and without radiation

Figures 2 was obtained for fins with height $h = 10mm$ and thickness $\delta = 1mm$. The meshes consisted of 50 x 50 nodes under surrounding temperature of $T_\infty = 300K$ and primary surface temperature of $T_b = 500K$.

Analyzing the results obtained and comparing them in Fig. 2, is evident the relevance of the phenomenon of radiation in extended surfaces, being clear that realistic experiments should not neglect this phenomenon.

3.3 Dimensionless parameters

This item will take a brief analysis of item 3.2 taking into account dimensionless parameters in order to generalize the verified observations.

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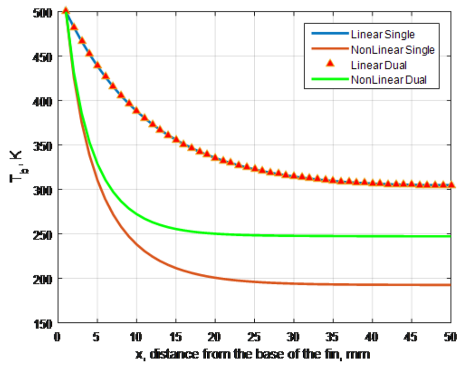

Figura 2: - Single and dual fins with and without radiation.

For N number of nodes, k thermal conductivity, C_1 and C_2 convection and radiation constants respectively, we have that the variation of the dimensionless temperature and the dimensionless position along the fin will have particularities according to the γ parameter showed in Fig. 3, that is, the relation between height and distance between the fins. Fig. 3 shows that when this relationship tends to infinity the interaction between the fins is despicable, for analytical aims there is no interaction between fins, accordingly considering single fin. U_1, U_2 and U_∞ being the local temperatures and ambient temperature. In Fig. 3 shows the relationship between dual and single fin temperatures with dimensionless position, attempting to the maximum point in this curve.

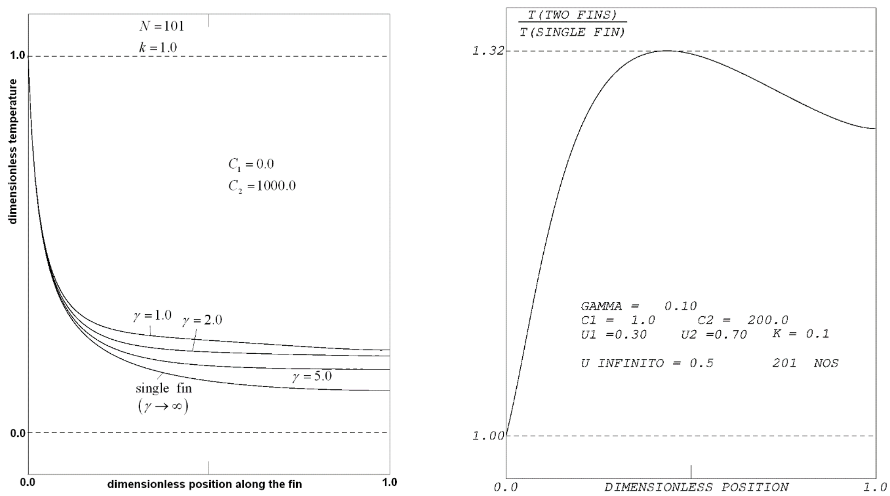

Figura 3: - Comparison and relation of dimensionless temperatures with position.

Being U_1, U_2 and U_∞ respectives local temperatures and environment temperature.

The Fig. 3 denotes a relation between dual and single temperatures fins varying with the dimensionless position, it is observed that there is a maximum point for such analysis.

4 Conclusions

The knowledge of the actual thermal mapping conditions plays a key role for an optimized design. A considerable methodology was presented in the present study, using theoretical, analytical and numerical treatment for finned surfaces. Results have shown both the relevance of the radiation and the heat mutuality so that there is an effective and realistic mapping.

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