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Increasing the Efficiency of the Brazilian Power Grid

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Abstract. We assess four methods that aim at improving the efficiency of electric power grids through the addition of new transmission lines, using complex network theory. We modeled the Brazilian high voltage transmission network as a graph in order to test these methods. The first method consists in connecting the pairs of nodes with the lowest degree in the network. The second connects the nodes with the lowest betweenness. The third links the pairs of nodes with the lowest degree that are connected to the mostly loaded nodes in the network. The last method connects nodes with median betweenness. Each method was tested with and without the *min-cut* procedure, which is capable of identifying the edges that, once removed, divide the network in two separate sub-networks. All the methods were capable of increasing the efficiency, but the strategy of connecting nodes with median betweenness with the support of *min-cut* was the most successful. All proposed strategies can be used to improve the efficiency of power networks. Furthermore, as these approaches are general, their application scope is not limited to power transmission lines.

Key Words. Complex networks, efficiency, topological analysis

1 Introduction

The large number of blackouts in recent years and the growing application of complex networks theory prompted academic research on the topological weaknesses of power grids. The topological arrangement of the networks is taken as part of the problem and some researchers have used complex network theory to characterize specific power grids such as the USA [1–4] , the Italian [5, 6], and three european networks [7]. In these analysis, the properties that can identify vulnerability points were calculated and the resistance was tested by removing transmission lines and substations, allowing the simulation of attacks and failures. They showed that the power lines were resistant to random failures, such as the fall of a transmission line or an arbitrary substation. However, the networks were fragile against attacks and disruptions [2, 3, 5, 7–9].

Efficiency in this work is defined as *the sum of the inverse of all shortest path lengths in the network*. In short, the shorter the edges in the network, the more efficient it is. Therefore, efficiency increases as the edges decrease in length [10] (Section 2). From the hypothesis that it is possible to improve the efficiency of a power network by means of topological changes, in this work we analyzed four approaches for adding new transmission lines to the system. We employed a graph-based model and applied complex network theory in order to test the increase in efficiency.

There are few studies that tackle the Brazilian case and its topological properties. The Brazilian network (which we call **BrNet**) has distinct features than the ones analyzed in the literature due to the relatively large geographical coverage and the fact that most of the hydroelectric power plants are distant from the distribution centers. This leads to power networks with long lines and few branches [11]. Consequently, the model employed in this work has nodes with fewer edges and nodes with a larger concentration of stations (e.g. Southeastern Brazil).

In this work, new edges were added to the network model of the BrNet according to four distinct strategies and we analyzed the increase in efficiency. Our goal was to test strategies that may aid the planning of new power lines, assessing existing methods and testing new heuristic capable of increasing the efficiency of complex networks by the addition of new edges. This work was carried out by means of a software tool specifically designed for the purpose of performing simulations.

Affonso et al. [12] have evaluated the BrNet stability using real operation and planning data and computed the stability threshold from 4 country regions, realizing that network stability is mainly related to the Southeast and Midwest regions. They examined the stability threshold in São Paulo, which was below the recommendations in some periods of the day. They assessed the transmission lines loss between the substations of Aparecida and Taubate (230 kV) and between Araraquara and Santa Barbara (440 kV) for the time the stability was below the threshold. The loss of the 230 kV line could cause problems, leading to more stability threshold reduction than the loss of the 440 kV line. Nevertheless, they highlighted that the 230 kV line could request local physical reinforcements for its recovery. Unlike the 230 Kv line, the loss of the 440 kV line would be more severe as it would require distributed maintenance in many system points to recovery its stability. Losing any of the two lines would not cause a breakdown, but could leave the system vulnerable. Hence, the BrNet stability is low and it is bounded by a specific region, suggesting that the topology is related to it. The BrNet lacks research providing pointers for improvements, as we propose in this work.

The remainder of this paper is organized as follows: In Section 2 we introduce the methodology and Section 3 addresses the results. In Section 4 we discuss the remarks and in Section 6 we present our conclusions.

2 Methodology

The data set used to build the power network model was obtained through the SINDAT system [1], which is made available by the Electric System National Operator (ONS). It consists of the record of electric power lines, power stations and substations represented in text format.

Our model ended up with 737 nodes and 1123 edges (not considering the redundant edges). The network data set employed is the one from May 2013. In this work we opted for developing specific *scripts* to run all the simulations and calculations needed. It was necessary to use the Beowulf Cluster from the School of Technology (Unicamp). In order

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[1] www.sindat.com.br

to speed up the testing phase, these *scripts* were created to maximize the performance by using the cluster's multiprocessing capabilities. All the work was carried out using Ubuntu-Linux and Python [2].

Efficiency is formally defined according to [10], and given by:

$$E = \frac{1}{N(N-1)} \sum_{i,j\epsilon\mathbb{N}} \frac{1}{d_{ij}} \tag{1}$$

where d_{ij} is the shortest length between node i and node j. The increase in efficiency was calculated using the Global Efficiency ($E(G)$). It was calculated each iteration for load distribution thus showing the gradual change in efficiency. We used the same definition and equations as Crucitti [1].

We simulated four strategies for adding edges to the network:

1. *Lowest Degree (LDg)*: This strategy was proposed by Zhao and Xu [13] and it consists in connecting the nodes with lowest degree in the network. Note: We slightly changed this strategy to ignore nodes with degree 1, as in the BrNet, due to geographical constraints, there are many nodes within this category. They mostly represent power plants that are far from their consumer centers.

2. *Lowest Load (LLo)*: This strategy, similar to LDg, connects the nodes with the lowest *betweenness* in the network. [3]

3. *Neighboring Nodes (NNo)*: This strategy works as follows: we find the 2 most loaded nodes in the network and save the nodes belonging to it. For each node, we find the neighbor with lowest degree. We then add an edge between each of these neighbors. The goal is to create an alternative path, thus distributing the load of the edges with larger load.

4. *Median Load (MLo)*: The last strategy consists in classifying the nodes by their load and finding out two median nodes.

The latter three strategies constitute original work.

In order to verify how relevant was the increase in efficiency due to the proposed strategies, we applied the following procedure:

1. Added a network edge, calculated the global efficiency, and stored the value;

2. Removed the added link;

3. Repeated steps 1 and 2 for all possible edges to be added;

4. Found which edge caused the largest gains in efficiency, and added them to the network;

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[2] http://www.python.org/

[3] It does not consider those with value 0 that are not intermediate nodes of any shortest path in the network. As there are many nodes (295) with load 0, the use of this method would result in a random method.

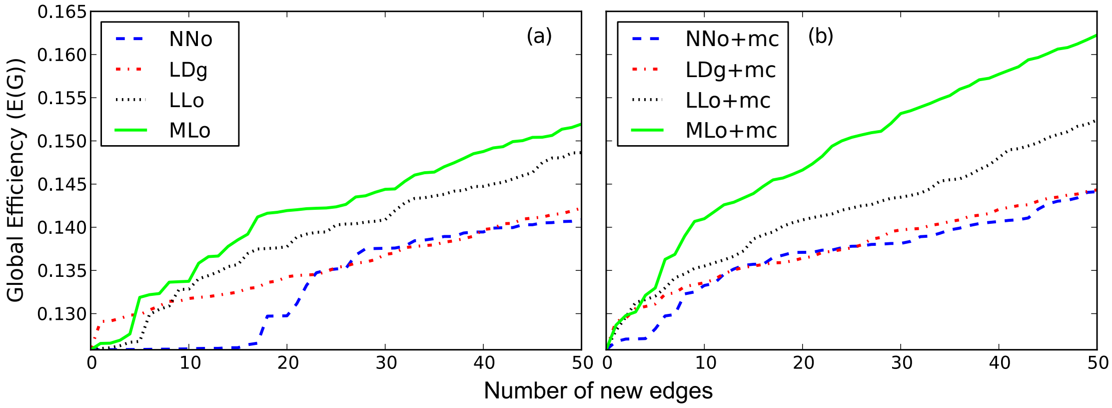

Figura 1: *Increase in efficiency for 50 added edges without min-cut (a) and with it (b).*

5. Using a new network, repeated the procedure until the network reaches 50 new edges.

This procedure calculates the largest possible theoretical gain in efficiency in the system. It is deemed to be a supporting strategy, as it serves as a reference value for comparing the major strategies and it is referred to as Optimal Edges (OEd). The idea of the MLo strategy follows from the observations that OEd suggested the connection of nodes with properties (e.g. degree and betweenness) around the median (or average). The betweenness metrics was used in the simulation of the load on the transmission lines and substations. It calculates how many small paths use a node or edge as an intermediate.

All strategies were tested with and without *min-cut*, as proposed by Rosato [7]. Since all strategies identify two nodes to be connected, when a strategy is combined and executed with *min-cut* each node is selected from a distinct subgraph. This contrasts with the procedure without *min-cut*, where the two nodes selected are always from the same connected graph. When using *min-cut*, the logic behind each strategy is not changed, it is just applied separately for each subgraph. To identify an strategy with *min-cut*, we add a "+mc"at the strategy acronym.

3 Experiments and Results

The first step of our experiment consisted in generating new networks from the BrNet model by adding 50 edges for each proposed strategy. Therefore, eight new networks were created from 4 strategies with and without the support of the *min-cut* procedure.

Once all the networks were generated and stored, we calculated the increase in efficiency obtained with each strategy. The efficiency was evaluated for each new added link, in order to follow the increase of efficiency in each network. We can observe in Fig. 1 that the largest increase in efficiency occurred when the strategies are supported by the *min-cut* (b). Notice that MLo presented the best results. Furthermore, the same strategies were successful in both cases (by observing the graphs individually).

We compared MLo and MLo+mc with the OEd method, as illustrated in Fig. 2. Notice that there was a significant difference, i.e. a 47.17 % increase in efficiency with

OEd compared to MLo+mc. However, OEd is unfeasible for real applications.

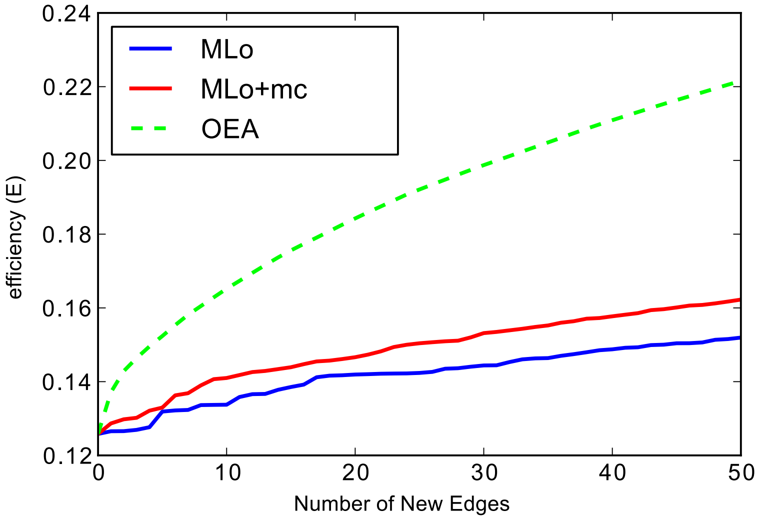

Figura 2: Comparison of MLo, MLo+mc and OEd. The MLo increased the final efficiency by 20.77%; MLo+mc showed an increase by 28.96% whereas OEd increased the efficiency by 76,10%.

4 Discussion

In this section we discuss the results from two different perspectives: 1) related to the BrNet, by considering the pros and cons in adopting the strategies outlined in this work; 2) from the complex network outlook, checking in which cases the strategies can be adopted in more general networks.

The idea of improving the network topology was proposed elsewhere with a different approach. The LDg strategy and the *min-cut* were used by other authors. This work adopted that as the starting point and attempted to search for other methods/strategies that would be also capable of changing the topology as a means to improve network robustness. Simulation has shown that LDg is the strategy that less contributed to the efficiency in all scenarios (Fig. 1).

After testing the LDg, NNo and LLo strategies, we searched for the optimal edges (OEd). This latter procedure is computationally heavy, leading to a 30-day processing using five machines with four Xeon 2.5 Ghz processors.

Although the increase in efficiency was much larger with the OEd strategy than with the other strategies, we noticed that all the edges obtained with OEd had one node in common. The resulting topology resembled that of a star. In regards to real-world power networks, this is not feasible, since one single power station would not accommodate such a large number of outgoing power lines. Another drawback was that, as shown by Crucitti [1], the loss of a heavily loaded node would have a large impact in the network resilience. It would be too risky to maintain a node with 20 or more edges in a power network, since its loss would imply in a total system collapse.

Even though OEd has shown to be inadequate to power networks, the increase in efficiency arose the question of what would be the criterium for choosing the nodes to connect a new edge and bring about an increase in efficiency. By observing the properties of the chosen nodes by 50 edges (i.e. *betweenness*, degree and cluster coefficient), the choice seemed to be around the average values. This prompted the idea of the MLo strategy, which outperformed the other strategies in regards to efficiency. It is possible to note that the majority of edges do not have nodes in common and much more sparsely laid around the graph. Although we did not analyze the geographical position of nodes in order to find out the actual length of the lines implied by the strategy, the fact that it does not concentrate the load in one single node makes it more prone to allow the insertion of more power lines in the system.

Additionally, we included the *min-cut* procedure to the strategies as a method capable of avoiding that the network is divided in components, allowing an increase in the network robustness. All the strategies showed an increase in efficiency when combined with this method.

Although OEd guarantees the best efficiency, its calculation time grows exponentially in relation to the number of nodes. For example, the power network had 739 nodes and it took around 30 days to execute. Clearly, it is an alternative to be considered when the algorithm performance is not an issue.

The MLo strategy can be used when we wish to increase the efficiency avoiding the edges to be concentrated around one node, and when the maximum increase in efficiency is not required. The execution of MLo in a single core 2.2 Ghz processor took around 2 seconds in the BrNet. Its execution time is proportional to the number of edges.

5 Summary and Conclusions

All four strategies were successful in increasing efficiency. Despite the fact that OEd offers the largest increase in efficiency of all strategies, in practice this strategy is unfeasible due to the fact that it is not possible to have all lines of a power network connected to a single node. Therefore, the MLo technique yielded the best results, by increasing the efficiency and maintaining the edges with few common nodes among each other.

We concluded that the MLo strategy can be successful in aiding the installation of new power lines, and due to the fact that its application is not cumbersome. Nevertheless, it should be combined with other methods that may guarantee with larger confidence that the lines will actually benefit the network. The strategies based on addition of new edges increased the efficiency in all tests.

As future work, we must consider the fact that the distances (and the real obstacles) of the lines indicated by the strategies were not the focus of this work. Further research that bounds the maximum distance between substations is required to avoid connections that are not feasible for some reason (economical or physical). Testing the resilience of the network against failures and attacks is also in our schedule, as this might show how our strategies change the resilience of the BrNet.

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