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Classic and Fuzzy Type-2 Control for the Van de Vusse Reactor: A Comparative Study

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A Proportional, Integral and Derivative controller with Ziegler-Nichols tuning for the Van de Vusse reactor, whose circuit includes a fuzzy rule-based system of interval type-2, is compared with the classic control. In the nonlinear process with inverse response of the Van de Vusse reactor, it is assumed that it is perfectly agitated and is operating in isothermal and isochoric conditions with constant reaction. The mathematical model that describes the reaction is given by a nonlinear system of ordinary differential equations which, by the Laplace transform method, is associated with a second order transfer function. Simulations built on Simulink ® show the efficacy of the interval type-2 fuzzy control strategy, with better performance in terms of delay and overshoot compared to the classic control.

Keywords. Van de Vusse reactor, Ziegler-Nichols tuning, Fuzzy Rule- Based System of interval type-2.

1 Introduction

Chemical reactors are one of the most important components of the chemical industry. A major problem in the chemical industry is that it is only possible to obtain the desired product from a reaction to prevent parallel reactions from occurring, so that the reaction obtained prevents parallel reactions from occurring, and the reaction yields its maximum. For this purpose, a way has been created to optimize processes through controllers, among which, the most widely used in the last two decades, is the Proportional, Integral and Derivative Control (PID).

The standard PID controller is also known as the three-term controller, whose transfer function is given by Equation (1)

$$H(s) = k_P + k_I\frac{1}{s} + k_D s = k_P\left(1 + \frac{1}{T_I s} + T_D s\right) \tag{1}$$

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where k_P is the proporcional gain, k_I is the integral gain and k_D is the derivative gain, T_I is the constant time of the integral, and T_D is the constant time of the derivative [1].

Van de Vusse reactor is a system with inverse response that occurs when the initial response of the output of the process is in the opposite direction of the steady state value. This reactor is isothermal, that is, of constant temperature. The reactor volume is also considered constant and the feed stream contains only the A component. The Van de Vusse reaction scheme consists of the irreversible reactions as in Equation (2):

$$\underbrace{A \xrightarrow{k_1} B \xrightarrow{k_2} C}_{(R_1)} \qquad \underbrace{2\,A \xrightarrow{k_3} D}_{(R_2)} \tag{2}$$

where A is the ciclopentadieno reagent, B, ciclopentenol is the desire product of the reaction. Due to the strong reactivity of the reagents A and B, they produce an undesired product, dicyclopentadiene, D, and a consecutive product, cyclopentanediol, C. The other components are: k_1, constant rate reaction of $A \xrightarrow{k_1} B$ in (R_1), k_2 a constant rate of the reaction $B \xrightarrow{k_1} C$ in (R_1), k_3 is the constant rare of reaction of (R_2). The reactor is perfectly agitated and is operating under isothermal and isochoric conditions with constant reaction. An illustration of the reactor is shown in Figure 1, where C_{A_0} is the initial concentration of A, C_A is the concentration of A, C_B is the concentration of B, F the density, and V the volume.

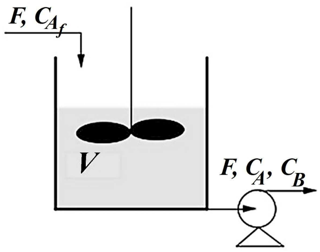

Figure 1: Illustration of the Van de Vusse reactor [2].

The objective of this research is to establish, from the dynamic model of the Van de Vusse [3] reactor, a control that includes a Fuzzy Rule-Based System (FRBS) of interval type-2 and compare it to the conventional PID control type. This controller is constructed from an FRBS in which the fuzzification is done through Mamdani's inference method on interval type-2 fuzzy sets [4], and the defuzzification is the generalized centroid method, obtained by the algorithm of Karnik and Mendel [5].

Simulations are done in Simulink ® software where PIDs are compared in terms of delay and overshoot indexes.

The paper is divided into five sections: Section 1 is the introduction; In Section 2 the transfer function of the Van de Vusse reactor is shown; in Section 3 a brief review of the basic concepts of fuzzy sets type-2 is made; in Section 4 the simulation of the classic PID compared with the controller that has FRBS of interval type-2 is simulated. Finally, Section 5 summarizes the conclusion.

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2 Van de Vusse Transfer Function

The initial concentration considered is $C_{A_0} = 10\text{gmoll}^{-1}$ and the volume V constant. Considering the equation of material balance and the fact that the volume is constant, we get the relation described in Equation (3)

$$\frac{d(V\rho)}{dt} = F_0\rho - F\rho, \tag{3}$$

where F_0 is the initial density and ρ could take values of the concentrations C_A, C_B, C_C, and C_D. Let us suppose that $F = F_0$, that is, a constant density. Denoting $\dfrac{F}{V} = G$, the Equation (3) appplied to each concentration [6], yields in the system of ordinary equations of Equation (4):

$$\begin{aligned} \frac{dC_A}{dt} &= G(C_{A_0} - C_A) - k_1 C_A - k_3 C_A^2, \\ \frac{dC_B}{dt} &= -GC_B + k_1 C_A - k_2 C_B, \\ \frac{dC_C}{dt} &= -GC_C + k_2 C_B, \\ \frac{dC_D}{dt} &= -GC_D + \frac{1}{2} k_3 C_A^2. \end{aligned} \tag{4}$$

The formation rate, r_A, in moll^{-1} of the component A is given by $r_A = -k_1 C_A - k_3 C_A^2$, and the formation rate, r_b, in moll^{-1} of the component B is $r_B = k_1 C_A - k_2 C_B$. Determining the state point of the first two equations of Equation (4), we obtain the expressions given in Equation (5)

$$C_{A_s} = \frac{-k_1 - G_s + \sqrt{(k_1 + G_s)^2 + 4k_3 G_s C_{A_0}}}{2k_3} \quad \text{and} \quad C_{B_s} = \frac{k_1 C_{A_s}}{G_s + k_2}, \tag{5}$$

where $G_s = \dfrac{F_s}{V} = \dfrac{F_0}{V}$. The nonlinear system o f Equation (4) can be approximated by the linear system in the matrix form given by Equation (6)

$$\dot{x} = Px + Qu, \qquad y = Rx + Su, \tag{6}$$

where the state variable, $x = \begin{bmatrix} C_A - C_{A_s} \\ C_B - C_{B_s}, \end{bmatrix}$, the input variable, $u = G - G_s$, and the output variable, $y = C_B - C_{B_s}$. Considering $\phi_1(C_A, C_B, G) = \frac{dC_A}{dt}$ and $\phi_2(C_A, C_B, G) = \frac{dC_B}{dt}$, the entries, p_{ij}, of the matrix P, and the entries, q_{ij}, of the matrix Q of the linear system (6) are calculated in Equation (7) as

$$\begin{aligned} p_{i1} &= \frac{\partial\phi_i}{dC_A}(C_{A_s}, C_{B_s}, G_s) \\ p_{i2} &= \frac{\partial\phi_i}{dC_B}(C_{A_s}, C_{B_s}, G_s) \\ q_{i1} &= \frac{\partial\phi_i}{dG}(C_{A_s}, C_{B_s}, G_s), \end{aligned} \tag{7}$$

for $i = 1, 2$. As a consequence, we obtain Equation (8)

$$P = \begin{bmatrix} -G_s - k_1 - 2k_3 C_{A_s} & 0 \\ k_1 & G_s - k_2 \end{bmatrix}, \; Q = \begin{bmatrix} C_{A_0} - C_{A_s} \\ -C_{B_s} \end{bmatrix},$$

$$R = [0 \;\; 1] \text{ and } S = 0. \tag{8}$$

The values of the constants for the reaction (R_1)-(R_2) considered in the simulation are $k_1 = 0.83 \text{ min}^{-1}$, $k_2 = 1.66 \text{ min}^{-1}$ and $k_3 = 0.166 \text{ moll}^{-1}$. The values of the steady states $C_{A_s} = 3\text{g mol l}^{-1}$, $C_{B_s} = 1.117\text{gmol l}^{-1}$, and $G = 0.5714 \text{ min}^{-1}$. All these values extracted from the literature [2,3,7]. Therefore, the matrix P and Q are numerically represented by the Equation (9)

$$P = \begin{bmatrix} -2.4 & 0 \\ 0.83 & -2.23 \end{bmatrix} \quad \text{and} \quad Q = \begin{bmatrix} 7 \\ -1.117 \end{bmatrix}. \tag{9}$$

Denoting $X(s)$, $Y(s)$, and $U(s)$ the Laplace transforms [8] of $x(t)$, $y(t)$, and $u(t)$, respectively, then the linear system (6), becomes Equation (10)

$$(sI - P)X(s) = QU(s), \quad Y(s) = RX(s). \tag{10}$$

Consequently, we have $Y(s)$ expressed as in Equation (11)

$$Y(s) = \left(R(sI - P)^{-1}Q\right) U(s). \tag{11}$$

Using the numerical matrix P and Q (9), the transfence function [8] of the model is given by the expression of Equation (12)

$$\frac{Y(s)}{U(s)} = \frac{-1.117s + 3.129}{s^2 + 4.63s + 5.352}. \tag{12}$$

The transference function in Equation (12) for the error control of the Van de Vusse reactor, is used in the simulations of Section 4.

3 Interval Type-2 Fuzzy Sets

The fuzzy sets of interval type-2 are an extension of the classical concept of fuzzy sets of type-1. In general, a fuzzy set of type-2, $\widetilde{A}$, is the graphic of a membership function $\mu_{\widetilde{A}} : X \times [0,1] \to [0,1]$. A fuzzy set of type-2 that verifies $\mu_A(x,u) = 1$ for all $(x,u) \in X \times [0,1]$ is denominated as interval fuzzy set of type-2 and it is totally determine for its footprint of uncertain [9]. A Fuzzy Rule-Based System (FRBS for short) of interval type-2 is an inference system that has at least one interval type-2 fuzzy set as an antecedent or as a consequent in the fuzzy inference [5]. The inference method used in this research is the Mamdani's method for interval type-2 fuzzy set [10].

4 Comparison of PID controllers

In this section the controller PID, classic and interval type-2 are compared. The Van de Vusse reactor's transfer function (12) is used to create a circuit that describes the control process, built through the Simulink® software. The input function of the circuit is the step function with of height G_s. At the end of the circuit, the output value o is subtracted to the input value, calculating in that way the error of the control process. This error has a gain, $k_P = 0.2$, which is proportional to the error itself. At the same time, the derivative of the error is calculated, which has as a gain $T_D = 0.95$, proportional to the rate of the error. Finally, the integral of the error is added with a proportional parameter $T_I = 0.23$. These parameters are extracted from the literature $[2, 3, 7]$. The total calculation value is the input for the transfer function, that represents the reactor in the process, finalizing with the output of the process. The circuit built in the Simulink environment is shown in Figure 2.

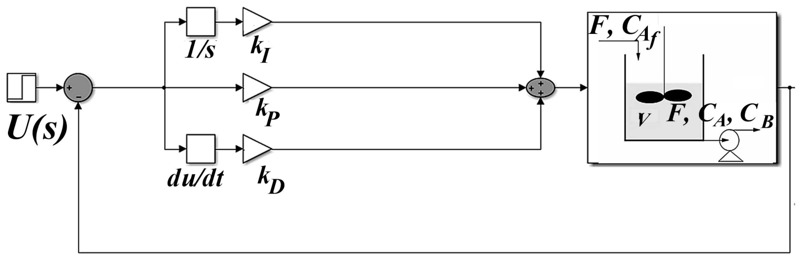

Figure 2: PID circuit controller in Simulink. The mathematical used in the control are extracted from the Simulink library. The "mask" for the transference function of the reactor is the same as the Figure 1.

In the controller that has the FRBS of interval type-2, the error, e, and the derivative of the error, e', enter in the control system through the inference of interval type-2, and the fuzzified value is add to the integral error value, adjusted by the same parameter T_I. The FRBS of interval type-2, is applied through a toolbox elaborated by Castillo [10].

The fuzzy set interval type-2, linguistic component of the input, are shown Figure 3 (a). The FRBS output has three linguistic components N (negative), P (positive) e Z (zero), which footprint of uncertainty are shown on the left of Figure 3 (b).

The established rules for the FRBS of interval type-2 are shown in Table 1.

Table 1: The rules for the FBRS of interval type-2.

e \ e'	N	P
N	N	Z
P	Z	P

The stability of the outputs of these values can be visually compared with the classic PID, and confirmed numerically by Table 2.

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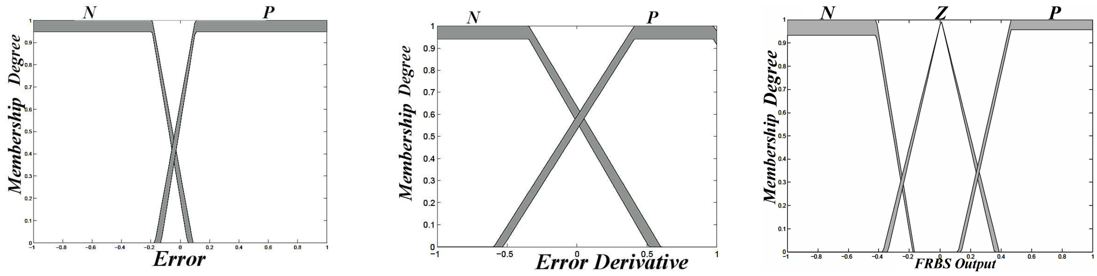

(a) Footprints of uncertainty of the error and error's derivative.

(b) Footprint of uncertain of the FRBS's output.

Figure 3: Input and output of the FRBS of type-2.

The band obtained from the defuzzification generated by the inference method of interval type-2 is shown together with the PID controller in Figure 4.

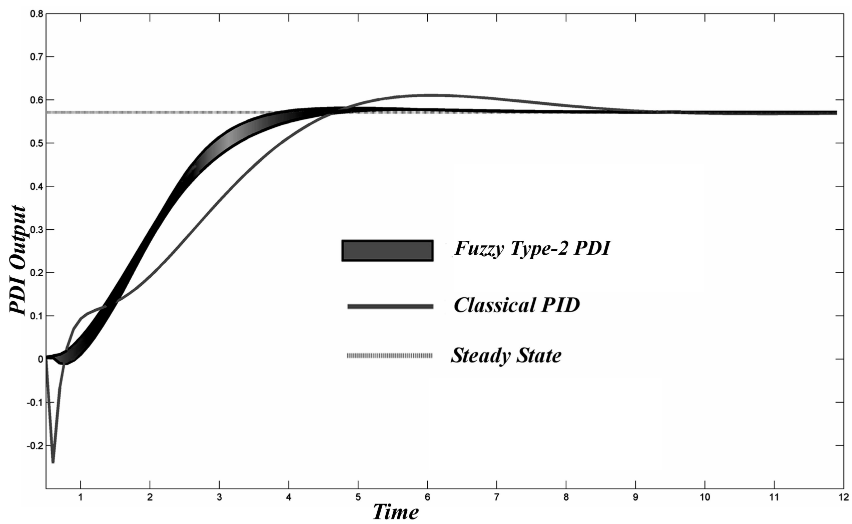

Figure 4: The PID controller and the type-2 band in time.

The numerical results allow a comparison in terms of two indexes:

- the number t_d, time delay that is the time necessary to the response get next to 50% of the steady state;
- the number M_{pt} which represents the overshoot corresponding to time t_p; the lesser the values M_{pt} and t_p, the faster the steady state is reached.

The comparative data is shown in Table 2, where the first and third columns are expressed in minutes.

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Table 2: Comparative table of the controllers.

	t_d	M_{pt}	t_p
Interval type-2 PID	2	0.5791	5.3
Classic PID	2.6	0.6107	6.1

5 Conclusion

A Proportional, Integral, and Derivative classic controller, attuned with Ziegler-Nichols parameters, is compared with a same type of controller using a Fuzzy Based-Rule System of interval type-2. The fuzzy controller of interval type-2 proved to be more efficient than the classical controller in the most important aspects of the output control behavior: the delay and the overshoot times. Simulations built in Simulink ® are developed in order to obtain the comparative numbers and graphics of the output control.

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