Output Feedback Control in Descriptor System Admissible

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Resumo. In this paper, we propose new techniques for synthesis of output feedback controllers of descriptor system to attain the closed loop admissibility. A necessary and sufficient condition is shown for the existence of such a controller in terms of linearized LMIs whose solution yields a controller that satisfies the specification. Considered the existence problem and the compute of output feedback that stabilizable in descriptor systems. This paper shows that based the coupled Sylvester equations and the coupled Lyapunov like equations can also be used to obtain a set of necessary and sufficient conditions for the existence of a output feedback controller, such that closed-loop system is admissible.

Palavras-chave. Output Feedback Control, Descriptor System,

1 Introduction

This paper deals the problem of stabilization by static output feedback for descriptor systems. Remind that an n-dimensional descriptor system consists of a mixture of n-q algebraic equations and q first order differential equations. Descriptor systems arise naturally in modelling of several dynamical systems commonly used engineering applications such as biological system, power systems and other interconnected systems [7], [10].

In the available literature on descriptor systems, there are two kinds of stabilization problems for singular continuous-time systems. One consists in designing a output-feedback controller in such a way that the closed-loop system is regular, impulse-free, and stable or equivalently admissible. The other is to design a output feedback controller in order to make the closed-loop system regular and stable.

The problem of computing a suitable static output feedback, from which these closed-loop properties are verified. These three desired properties can be described in terms of the closed-loop eignestructure: (i) the asymptotic stability is equivalent to have all the finite
poles in the left half complex plane; (ii) the absence of impulsive modes is equivalent to have q finite closed-loop; and (iii) the regularity is guaranteed if the system is impulse-free. Thus the necessary and sufficient conditions for the existence of a stabilizing output feedback are obtained as a set of coupled (generalized) Sylvester equations in [8], [9], [4], [5] [14].

This paper is organized as follows. The second section, presents the problem, based in the basic concepts, the necessary and sufficient conditions for existence of a solution. In section 3 presents some results main with formulation the theorems. In section 4 presents the aspects algorithmic and presents The numerical example illustrate the application of the algorithm that outlines the basic steps are used to solve the problem is presented in the fourth section. In the section 5 presents the regulator problem. Finally, concluding remarks are apresented.

2 Preliminaries

The considered linear descriptor systems are described by:

\[ E \dot{x}(t) = Ax(t) + Bu(t) \]  
\[ y(t) = Cx(t) + Du(t) \] (1)

where: \( x \in \mathcal{X} \sim \mathbb{R}^n \), \( u \in \mathcal{U} \sim \mathbb{R}^m \), \( y \in \mathcal{Y} \sim \mathbb{R}^p \), and \( E \in \mathbb{R}^{n \times n} \), \( \text{rank}(E) = q < n \); as the other matrices is an appropriate size with \( \text{rank}(B) = m \), \( \text{rank}(C) = p \) and \( \text{rank}(D) = m \). The pair \((E,A)\) is called regular if there exists \( s \in \mathbb{C} \) such that \( \det(sE - A) \neq 0 \). Thus a regular descriptor system is in [13] and [7].

i) stable if all finite roots of \( \det(sE - A) = 0 \) are in the open left half complex plane;
ii) impulse free if it exhibits no impulse behavior;
iii) finite dynamics detectable if there exists \( L \) such that \((E, A + LC)\) is regular and stable;
iv) impulse observable if there exists \( L \) such that \((E, A + LC)\) is regular and impulse-free.

Considered the descriptor system

\[ E \dot{x}(t) = Ax(t) \] (3)

Consider as follow definitions

**Definition 2.1.** [15] The system (1), (2) is said to be regular if \( \det(sE - A) \neq 0 \).

**Definition 2.2.** [15] The system (1), (2) is said to be impulse-free is \( \text{deg}(\det(sE - A)) = \text{rank}(E) \).

**Definition 2.3.** [15] The system (1), (2) is said to be stable if all the roots of \( \text{deg}(\det(sE - A)) = 0 \) have negative real part.

**Definition 2.4.** [15] The system (1), (2) is said to be admissible if it is regular, impulse-free and stable.

The objective the paper is to find a static output feedback controller

\[ u(t) = Gy(t), \] (4)

such that the closed-loop system is admissible.

\[ E \dot{x}(t) = (A + BGC)x(t), \]
\[ y(t) = Cx(t) \] (5)
Theorem 2.1. The system (1), (2) is admissible if and only if there exist a scalar $\epsilon > 0$ and matrices $\tilde{P} > 0$, $\tilde{U} > 0$, $\tilde{W} > 0$ and $\tilde{Q} > 0$ such that

$$
\begin{pmatrix}
-E'\tilde{P}E - \tilde{W} & \epsilon A' + E' & \epsilon A' & \tilde{Q}'S'
\end{pmatrix} < 0
$$

where $S \in \mathbb{R}^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E'S = 0$.

The presented a version of the generalized Lyapunov theorem in [11].

Theorem 2.2. [11] Let $(E,A)$ be regular and $(E,A,C)$ be impulse observable and finite dynamics detectable. Then $(E,A)$ is stable and impulse-free if and only if there exists a solution $(P,Q)$ to the equation:

$$
A'P + Q'A + C'C = 0; \quad Q'E = E'P \geq 0
$$

3 Main results

This paper show that based the coupled Sylvester equations and the coupled Lyapunov like equations can also be used to obtain a set of necessary and sufficient conditions for the existence of an output feedback controller $u = \epsilon^{-1}\tilde{K}y(t)$ $u = Gy$ with $G = \epsilon^{-1\tilde{K}}$ , such that closed-loop system (5) is admissible.

Theorem 3.1. Given the continuous descriptor systems (1), (2). There exists a static output feedback controller (4) such that closed-loop system (5) is admissible if there exist a scalar $\epsilon > 0$, matrices $\tilde{P} > 0$, $\tilde{U} > 0$, $\tilde{W} > 0$ and $\tilde{Q} > 0$ such that

$$
\begin{pmatrix}
-E'\tilde{P}E - \tilde{W} & \epsilon A' + E' & \epsilon A' & \tilde{Q}'S'
\end{pmatrix} < 0
$$

where $S \in \mathbb{R}^{n \times (n-r)}$ is any matrix with full column rank and satisfies $E'S = 0$. in this case, a desired static output feedback control law can be chosen as

$$
u(t) = \epsilon^{-1}\tilde{K}Cx(t) \quad u(t) = Gy(t)
$$

Remark 3.1. The Theorem (3.1) presents necessary and sufficient conditions for the admissibility of continuous descriptor systems (1), (2). The Theorem (3.1) provides a sufficient condition for existence of the output feedback controller for continuous descriptor systems.
The Theorem is presented as follows:

**Theorem 3.2.** If there exist a scalar \( \epsilon > 0 \), matrices \( \tilde{P} > 0 \), \( \tilde{U} > 0 \), \( \tilde{W} > 0 \) and \( \tilde{Q} > 0 \) such that the equations (10), (11) and (12) are satisfied. Then there exists an output feedback matrix \( G : \mathcal{Y} \rightarrow \mathcal{U} \) such that closed-loop system (5) is admissible. If the sufficient following conditions are verified for some positive scalar \( v \leq n \) and for some pair of matrices \( V \in \mathbb{R}^{n \times v} \) and \( T \in \mathbb{R}^{q-v \times n} \), such that \( TEV = 0 \), where \( \text{rank}(EV) = q \): 

(i) Let \( Q = C' \), \( \tilde{Q} \in \mathbb{R}^{n \times n} \), there exist matrices \( P = P' \geq 0 \), \( P \in \mathbb{R}^{n \times n} \) and \( Y = WV' \in \mathbb{R}^{m \times n} \), such that:

\[
APE' + EPA' = -EC'C'E' \tag{15}
\]

\[
Pd = V'PV > 0 ; \quad TEPE'T = 0 \tag{16}
\]

\[
Y = GCP \tag{17}
\]

The theorem (3.2) is propose numerically solution algorithm to determine the output feedback matrix in descriptor systems, where the equations (10), (11) and (12) are satisfied and such that closed-loop system (5) is admissible. The theorem (3.2) it is similarly at [2], [5].

4 Aspects algorithmic

Is shown below that based coupled Sylvester equations and the coupled Lyapunov like equations can also be used to obtain the sufficient conditions for the existence of an output feedback, such that closed-loop system (5) is admissible.

4.1 The Syrmos-Lewis algorithm

Considered the Theorem (3.1) where a scalar \( \epsilon > 0 \), matrices \( \tilde{P} > 0 \), \( \tilde{U} > 0 \), \( \tilde{W} > 0 \) and \( \tilde{Q} > 0 \) such that the equations (10), (11), (12) are satisfied. Then for the calculation of the output feedback that stabilizes the closed-loop system, when \( m + p > q \).

Therefore determined The Syrmos-Lewis algorithm the existence of an output feedback, such that closed-loop system (5) is admissible.

4.2 Example

Considered the following data [6]:

\[
E = \begin{bmatrix}
1.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 1.00 & 1.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 0.00
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
0.00 & 1.00 & 0.00 & 0.00 \\
0.00 & 0.00 & 0.00 & 1.00 \\
1.00 & 1.00 & 0.00 & 0.00 \\
0.00 & -1.00 & 1.00 & 1.00
\end{bmatrix}
\]
\[
B = \begin{bmatrix}
0.00 & 0.00 \\
0.00 & 0.00 \\
1.00 & 0.00 \\
0.00 & 1.00 \\
\end{bmatrix} ;
C = \begin{bmatrix}
1 & 0 & 0
\end{bmatrix}
\]

The corresponding descriptor system with finite poles are given by: \( \sigma(E, A) = \{0.0, 0.0, -1.0\} \). The system \((C, E, A, B)\) is both strongly controllable and observable.

In a first step, considered a scalar \( \epsilon > 0 \), matrices \( \tilde{P} > 0, \tilde{U} > 0, \tilde{W} > 0 \) and \( \tilde{Q} > 0 \) such that the equations (10), (11) and (12) in theorem (3.1) are satisfied.

In a second step, eigenstructure assignment is used thus the eigenvalues to positioned are given for:

\[ \Lambda_T = \{-1\} \cup \Lambda_V = \{-3.5\} \]

**Step 1:** For \( \lambda_1 = -1, t_j \in \mathbb{C}^n \) and \( u_j \in \mathbb{C}^p \), such that:

\[
\begin{bmatrix}
\begin{bmatrix}
0
\end{bmatrix} & \begin{bmatrix}
0
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
A - \lambda_j E
C
\end{bmatrix}
= 0 \quad \forall \ j = 1, \ldots, q - p
\]

Step 3: For \( \lambda_2 = -3.5 \),

\[
\begin{bmatrix}
A - \lambda_i E & B \\
TE & 0
\end{bmatrix}
\begin{bmatrix}
P_i \\
Y_i
\end{bmatrix}
= 0 \forall \ i = q - p + 1, \ldots, q
\]

Step 4: Determined \( G \) that such \( GCP = Y \):

\[
G = \begin{bmatrix}
2.5000 \\
-1.1923
\end{bmatrix}
\]

\[
A + BGC = \begin{bmatrix}
0.0 & 1.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 \\
3.5000 & 1.0 & 0.0 & 0.0 \\
-1.1923 & -1.0 & 1.0 & 1.0
\end{bmatrix}
\]

The corresponding closed-loop system \((E, A + BGC)\) has the desired generalized eigenvalues.

5 **Regulator Problem**

Consider the optimal regulator problem that minimizes

\[
J = \frac{1}{2} \int_0^\infty ||y(t)||^2 dt
\]
subject to (1), (2) and with
\[ \lim_{t \to \infty} x(t) = 0 \]  

We assume that \((E, A, B)\) be impulse controllable and finite dynamics stabilizable and that \((E, A, C)\) be impulse observable and dynamics detectable. Let \(G\) be a output feedback matrix such that \((E, A + BGC)\) is stable and impulse-free, such that closed-loop system (5) is admissible. Therefore if there exist a scalar \(\epsilon > 0\), matrices \(\tilde{P} > 0, \tilde{U} > 0, \tilde{W} > 0\) and \(\tilde{Q} > 0\) such that the equations (10), (11) and (12) in theorem (3.1) are satisfied.

Then from the symmetric version of Theorem (2.2), there exists a solution \(P\) to

\[
(A + BGC)'P + P'(A + BGC) + QQ + (GC)'SS' + SSGC + (CG)'R(CG) = 0; \quad E'P = PE \geq 0
\]

where \(QQ = C'C, SS = C'D, R = D'D > 0\). It can be shown that the performance index is expressed as

\[
J = \frac{1}{2} x_0'P'E'Px_0 \geq 0
\]

6 Concluding remarks

In this paper, we proposed new techniques for synthesis of output feedback controllers of descriptor system to attain the closed loop admissibility. A necessary and sufficient condition is shown for the existence of such a controller in terms of linearized LMIs whose solution yields a controller that satisfy the specification. Considered the existence problem and the compute of output feedback that strongly stabilizable in descriptor systems.

Referências


