Proceeding Series of the Brazilian Society of Computational and Applied Mathematics, v. 6, n. 2, 2018.

Trabalho apresentado no XXXVIII CNMAC, Campinas - SP, 2018.

Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Output Feedback Control in Descriptor System Admissible

Elmer Rolando Llanos Villarreal[1]
Departamento de Ciências Naturais, Matemática e Estatística,
Universidade Federal Rural do Semi-árido, 59625900, Mossoró, RN, Brasil.
Andrés Ortiz Salazar[2]
Departamento de Engenharia de Computação e Automação
Universidade Federal de Rio Grande do Norte,
59078-970, Natal, RN, Brasil.
Alberto Soto Lock[3]
Departamento de Engenharia Elétrica
Universidade Federal Paraiba, 58051-900, João Pessoa, PB, Brasil.
Walter Martins Rodrigues[4]
Departamento de Ciências Naturais, Matemática e Estatística,
Universidade Federal Rural do Semi-árido, 59625900, Mossoró, RN, Brasil.

Resumo. In this paper, are propose new techniques for synthesis of output feedback controllers of descriptor system to attain the closed loop admissibility. A necessary and sufficient condition are shown for the existence of such a controller in terms of linearized LMIs whose solution yields a controller that satisfy the specification. Considered the existence problem and the compute of output feedback that stabilizable in descriptor systems. This paper show that based the coupled Sylvester equations and the coupled Lyapunov like equations can also be used to obtain a set of necessary and sufficient conditions for the existence of a output feedback controller, such that closed-loop system is admissible.

Palavras-chave. Output Feedback Control, Descriptor System,

1 Introduction

This paper deals the problem of stabilization by static output feedback for descriptor systems. Remind that an n-dimensional descriptor system consists of a mixture de n-q algebraic equations and q firts order diferential equations. Descriptor systems arise naturally in modelling of several dynamical systems commonly used engineering applications such as biological system, power systems and other interconnected systems [7], [10].

In the available literature on descriptor systems, there are two kinds of stabilization problems for singular continuous-time systems. One consists in designing a output-feedback controller in such a way that the closed-loop system is regular, impulse-free, and stable or equivalently admissible. The other is to design a output feedback controller in order to make the closed-loop system regular and stable.

The problem of computing a suitable static output feedback, from which these closed-loop propierties are verified. These three desired properties can be described in terms of the closed-loop eigenstructure: (i) the asymptotic stability is equivalent to have all the finite

[1]elmerllanos@ufersa.edu.br
[2]andres@dca.ufrn.br
[3]aslock@cear.ufpb.br
[4]walterm@ufersa.edu.br

poles in the left half complex plane; (ii) the absence of impulsive modes is equivalent to have q finite closed-loop; and (iii) the regularity is guaranteed if the system is impulse-free. Thus the necessary and sufficient conditions for the existence of a stabilizing output feedback are obtained as a set of coupled (generalized) Sylvester equations in [8], [9], [4], [5] [14] .

This paper is organized as follows. The second section, presents the problem, based in the basic concepts, the necessary and sufficient conditions for existence of a solution. In section 3 presents some results main with formulation the theorems . In section 4 presents the aspects algorithmic and presents The numerical example illustrate the application of the algorithm that outlines the basic steps are used to solve the problem is presented in the fourth section. In the section 5 presents the regulator problem. Finally, concluding remarks are apresented.

2 Preliminaries

The considered linear descriptor systems are described by:

$$E\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$
$$y(t) = Cx(t) + Du(t) \quad (2)$$

where: $x \in \mathcal{X} \sim \Re^n$, $u \in \mathcal{U} \sim \Re^m$, $y \in \mathcal{Y} \sim \Re^p$ and $E \in \Re^{n\times n}$, $rank\,(E) = q < n$; as the other matrices is an appropriate size with $rank\,(B) = m$, $rank\,(C) = p$ and $rank\,(D) = m$. The pair (E, A) is called regular if there exists $s \in \mathcal{C}$ such that $det(sE - A) \neq 0$ Thus a regular descriptor system is in [13] and [7].

i) stable if all finite roots of $det(sE - A) = 0$ are in the open left half complex plane;

ii) impulse free if it exhibits no impulse behavior;

iii) finite dynamics detectable if there exists L such that $(E, A + LC)$ is regular and stable;

iv) impulse observable if there exists L such that $(E, A + LC)$ is regular and impulse-free;

Considered the descriptor system

$$E\dot{x}(t) = Ax(t) \quad (3)$$

Consider as follow definitions

Definition 2.1. *[15] The system (1), (2) is said to be regular if* $det(sE - A) \neq 0$.

Definition 2.2. *[15] The system (1), (2) is said to be impulse-free is* $deg(det(sE-A)) = rank(E)$.

Definition 2.3. *[15] The system (1), (2) is said to be stable if all the roots of deg(det(sE-A))=0 have negative real part.*

Definition 2.4. *[15] The system (1), (2) is said to be admissible if it is regular, impulse-free and stable.*

The objective the paper is to find a static output feedback controller

$$u(t) = Gy(t), \quad (4)$$

such that the closed-loop system is admissible.

$$\begin{aligned} E\dot{x}(t) &= (A + BGC)x(t), \\ y(t) &= Cx(t) \end{aligned} \quad (5)$$

Theorem 2.1. *The system (1), (2) is admissible if and only if there exist a scalar $\epsilon > 0$ and matrices $\tilde{P} > 0$, $\tilde{U} > 0$, $\tilde{W} > 0$ and $\tilde{Q} > 0$ such that*

$$\begin{pmatrix} -E'\tilde{P}E - \tilde{W} & \epsilon A' + E' & \epsilon A' & \tilde{Q}'S' \\ \epsilon A + E & -\tilde{U} & 0 & 0 \\ \epsilon A & 0 & -I & 0 \\ S\tilde{Q} & 0 & 0 & -I \end{pmatrix} < 0 \tag{6}$$

$$\tilde{W} < (\epsilon A' - \tilde{Q}'S')(\epsilon A - S\tilde{Q}), \tag{7}$$

$$\tilde{P}\tilde{U} = I \tag{8}$$

where $S \in \Re^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E'S = 0$.

The presented a version of the generalized Lyapunov theorem in [11].

Theorem 2.2. *[11] Let (E, A) be regular and (E, A, C) be impulse observable and finite dynamics detectable. Then (E, A) is stable and impulse-free if and only if there exists a solution (P, Q) to the equation:*

$$A'P + Q'A + C'C = 0; \quad Q'E = E'P \geq 0 \tag{9}$$

3 Main results

This paper show that based the coupled Sylvester equations and the coupled Lyapunov like equations can also be used to obtain a set of necessary and sufficient conditions for the existence of a output feedback controller $u = \epsilon^{-1}\tilde{K}y(t)$ $u = Gy$ with $G = \epsilon^{-1}\tilde{K}$, such that closed-loop system (5) is admissible.

Theorem 3.1. *Given the continuous descriptor systems (1), (2). There exists a static output feedback controller (4) such that closed-loop system (5) is admissible if there exist a scalar $\epsilon > 0$, matrices $\tilde{P} > 0$, $\tilde{U} > 0$, $\tilde{W} > 0$ and $\tilde{Q} > 0$ such that*

$$\begin{pmatrix} -E'\tilde{P}E - \tilde{W} & \epsilon A' + E' & \epsilon A' & \tilde{Q}'S' \\ \epsilon A + E & -\tilde{U} & 0 & 0 \\ \epsilon A & 0 & -I & 0 \\ S\tilde{Q} & 0 & 0 & -I \end{pmatrix} < 0 \tag{10}$$

$$\tilde{W} < (\epsilon A' - \tilde{Q}'S')(\epsilon A - S\tilde{Q}), \tag{11}$$

$$\tilde{P}\tilde{U} = I \tag{12}$$

where $S \in \Re^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E'S = 0$. in this case, a desired static output feedback control law can be chosen as

$$u(t) = \epsilon^{-1}\tilde{K}Cx(t) \tag{13}$$

$$u(t) = Gy(t) \tag{14}$$

Remark 3.1. *The Theorem (3.1) presents necessary and sufficient conditions for the admissibility of continuous descriptor systems (1), (2). The Theorem (3.1) provides a sufficient condition for existence of the output feedback controller for continuous descriptor systems.*

DOI: 10.5540/03.2018.006.02.0309

The Theorem is presented as follows:

Theorem 3.2. *If there exist a scalar $\epsilon > 0$, matrices $\tilde{P} > 0$, $\tilde{U} > 0$, $\tilde{W} > 0$ and $\tilde{Q} > 0$ such that the equations (10) , (11) and (12) are satisfied.Then there exists an output feedback matrix $G : \mathcal{Y} \longrightarrow \mathcal{U}$ such that closed-loop system (5) is admissible. if the sufficient following conditions are verified for some positive scalar $v \leq n$ and for some pair of matrices $V \in \Re^{n\times v}$ and $T \in \Re^{q-v\times n}$, that such $TEV = 0$, where $rank\,(EV) = rank\,(TE) = q$:*
(i) Let $Q = C'C$, $Q \in \Re^{n\times n}$, there exist matrices $P = P' \geq 0$, $P \in \Re^{n\times n}$ and $Y = WV' \in \Re^{m\times n}$, such that:

$$APE' + EPA' \;=\; -EC'CE' \tag{15}$$
$$Pd = V'PV > 0 \quad ; \quad TEPE'T' = 0 \tag{16}$$

$$Y \;=\; GCP \tag{17}$$

The theorem (3.2) is propose numerically solution algorithm to determine the output feedback matrix in descriptor systems, where the equations (10) , (11) and (12) are satisfied and such that closed-loop system (5) is admissible. The theorem (3.2) it is similarly at [2], [5].

4 Aspects algorithmic

Is shown below that based coupled Sylvester equations and the coupled Lyapunov like equations can also be used to obtain the sufficient conditions for the existence of a output feedback, such that closed-loop system (5) is admissible.

4.1 The Syrmos-Lewis algorithm

Considered the Theorem (3.1) where a scalar $\epsilon > 0$, matrices $\tilde{P} > 0$, $\tilde{U} > 0$, $\tilde{W} > 0$ and $\tilde{Q} > 0$ such that the equations (10), (11), (12) are satisfied. Then for the calculation of the output feedback that stabilizes the closed-loop system, when $m + p > q$.

Therefore determined The Syrmos-Lewis algorithm the existence of a output feedback, such that closed-loop system (5) is admissible.

4.2 Example

Considered the following data [6] :

$$E = \begin{bmatrix} 1.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix};$$

$$A = \begin{bmatrix} 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 1.00 \\ 1.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & -1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.00 \\ 1.00 & 0.00 \\ 0.00 & 1.00 \end{bmatrix} ; \; C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

The corresponding descriptor system with finite poles are given by: $\sigma(E, A) = \{0.0,\ 0.0,\ -1.0\}$. The system (C, E, A, B) is both stronlgy controllable and observable.

In a first step, considered a scalar $\epsilon > 0$, matrices $\tilde{P} > 0$, $\tilde{U} > 0$, $\tilde{W} > 0$ and $\tilde{Q} > 0$ such that the equations (10), (11) and (12) in theorem (3.1) are satisfied.

In a second step, eigenstructure assignment is used thus the eigenvalues to positioned are given for : $\Lambda_T = \{-1\} \cup \Lambda_V = \{-3.5\}$.

Step 1 : For $\lambda_1 = -1$, $t_j \in \mathcal{C}^n$ and $u_j \in \mathcal{C}^p$, such that :

$$\begin{bmatrix} t_j' & u_j' \end{bmatrix} \begin{bmatrix} A - \lambda_j E \\ C \end{bmatrix} = 0 \;\; \forall \;\; j = 1, \ldots, q - p \tag{18}$$

determined T that verified (18) and such that $TEP = 0$ is also verified and (A, B, T, E) have not invariant zeros :

$$T = \begin{bmatrix} 0.00 & 0.00 & 0.7071 & -0.7071 \end{bmatrix}$$

Step 3 : For $\lambda_2 = -3.5$,

$$\begin{bmatrix} A - \lambda_i E & B \\ TE & 0 \end{bmatrix} \begin{bmatrix} P_i \\ Y_i \end{bmatrix} = 0 \, \forall \; i = q - p + 1, \ldots, q \tag{19}$$

determined P and Y using (19) :

$$P = \begin{bmatrix} 0.1762 \\ -0.6167 \\ 0.1626 \\ -0.5693 \end{bmatrix} \;\; ; \;\; Y = \begin{bmatrix} 0.4405 \\ -0.2101 \end{bmatrix};$$

Step 4 : Determined G that such $GCP = Y$:

$$G = \begin{bmatrix} 2.5000 \\ -1.1923 \end{bmatrix};$$

$$A + BGC = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 3.5000 & 1.0 & 0.0 & 0.0 \\ -1.1923 & -1.0 & 1.0 & 1.0 \end{bmatrix}.$$

The corresponding closed-loop system $(E, A + BGC)$ has the desired generalized eigenvalues.

5 Regulator Problem

Consider the optimal regulator problem that minimizes

$$J = \frac{1}{2} \int_0^{\infty} ||y(t)||^2 dt \tag{20}$$

subject to (1), (2) and with

$$lim_{t \longrightarrow \infty} x(t) = 0 \tag{21}$$

We assume that (E, A, B) be impulse controllable and finite dynamics stabilizable and that (E, A, C) be impulse observable and dynamics detectable. Let G be a output feedback matrix such that $(E, A + BGC)$ is stable and impulse-free, such that closed-loop system (5) is admissible. Thefore if there exist a scalar $\epsilon > 0$, matrices $\tilde{P} > 0$, $\tilde{U} > 0$, $\tilde{W} > 0$ and $\tilde{Q} > 0$ such that the equations (10), (11) and (12) in theorem (3.1) are satisfied.

Then from the symmetric version of Theorem (2.2), there exists a solution P to

$$\begin{aligned}(A + BGC)'P + P'(A + BGC) + QQ + (GC)'SS' +& \\ SSGC + (CG)'R(CG) = 0; \quad E'P = PE \geq 0&\end{aligned} \tag{22}$$

where $QQ = C'C$, $SS = C'D$, $R = D'D > 0$. It can be shown that the the performance index is expressed as

$$J = \frac{1}{2} x_o' P' E' P x_0 \geq 0 \tag{23}$$

6 Concluding remarks

In this paper, we proposed new techniques for synthesis of output feedback controllers of descriptor system to attain the closed loop admissibility. A necessary and sufficient condition is shown for the existence of such a controller in terms of linearized LMIs whose solution yields a controller that satisfy the specification. Considered the existence problem and the compute of output feedback that strongly stabilizable in descriptor systems.

Referências

[1] P. Bernhard, On singular implicit linear dynamical systems, *SIAM Journal on Control and Optimization*, 20(5):612–633, 1982.

[2] E. B. Castelan and Elmer R. Llanos Villarreal and Sophie Tarbouriech, Quadratic Characterization and Use of Output Stabilizable Subspaces in Descriptor Systems, *Proceedings (CD) 1st IFAC Symposium on System Structure, Prague,2001*, vol. 1, 207–212, 2001.

[3] E. B. Castelan and J. C. Hennet and E. R. Ll. Villarreal, Quadratic Characterization and Use of Output Stabilizable Subspaces,*IEEE Trans. Automatic. Control*, 48(4):654–660, 2003.

[4] E. B. Castelan and J. C. Hennet and E. R. Llanos Villarreal, Output Feeedback Design by Coupled Lyapunov-Like Equations, *Proceedings of the* 15^{th} *Ifac Congress on the International Federation of Automatic Control*, vol. 35. p. 207–212,2002.

[5] E. B. Castelan, Estabilização de Sistemas Descritores por Realimentação de Saídas via Subespaços Invariantes, *Revista de Controle e Automação*, 16(4):467–477, 2005.

[6] D. L. Chu and H. C. Chan and D. W. C. Ho, Necessary and Suficient Conditions for the Output Feedback Regularization of Descriptor Systems *IEEE Trans. Automatic. Control*, 44(2):1457–1468, 1999.

[7] L. Dai, Singular Control System, Springer-Verlag, 1989.

[8] , G. R. Duan, Eigenstructure assignment in descriptor systems via output feedback: a new complete parametric approach, *Int J. Control*, 72(4):345–364, 1999.

[9] L. R. Fletcher, Eigenstructure Assingment by Output Feedback in Descriptor Systems, *IEE Proceedings*, vo135(4):302–308, 1998.

[10] F. L. Lewis, A Survey of Linear Singular Systems, *Circuits Systems Signal Process*, 5(1):3–36, 1986.

[11] K. Tabaka, N. Morihira and T. Katayama, A generalized Lyapunov theorem for descriptor system, *Systems* & *Control Letters*, 24:49–51, 1995.

[12] A. Varga, On stabilization methods of descriptor systems, *Systems* & *Control Letters*, 24(2):133–138, 1995.

[13] G. C. Verghese, B C. Levy, and T. Kailath, A Generalized State-Space for Singular Systems, *IEEE Trans. Automatic. Control* AC-26(4):811–831, 1981.

[14] E. R. Ll. Villarreal, J. A. R. Vargas, E. M. Hemerly, Static Ouput Feedback stabilization using Invariant Subspaces and Sylvester Equations, *TEMA - Tend. Mat. Apl. Comput.*, 10(1):99–110, 2009.

[15] S. Xu and J. Lam, Robust Control and Filtering of Singular Systems, *Lecture Notes in Control and Information Sciences, Springer* 1:11–29, 2006.

