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Mars climate engineering using space solar reflectors in Sun-synchronous polar orbits

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Abstract. Several space-based climate engineering methods, including shading the Earth with a particle ring for active cooling, or the use of orbital reflectors to increase the total insolation of Mars for climate warming have been proposed to modify planetary climates in a controller manner. In this study, solar reflectors on Sun-synchronous polar orbits (frozen orbits) normal to the ecliptic plane to the Mars are considered to intervene in the Mars's climate system.The two-body problem is considered, and the Gauss' form of the variational equations is used to describe the propagation of the polar orbit, taking into account the effects of solar radiation pressure and Mars's J_2 oblateness perturbation.

Key-words. Climate engineering, Space solar reflectors, Sun-synchronous polar orbits, Solar radiation, J_2 oblateness perturbation

1 Introduction

In the last forty years, geo-engineering schemes have been the subject of numerous studies for a possible futuristic use of orbiting solar reflectors for illumination-from-space applications, e.g. providing extra hours of illumination for energy supplies or terraforming schemes (engineering an Earth-like climate) [2, 4–6, 13]. The main advantage is the vast energy leverage delivered by the reflectors which is obtained in a relatively short time [9]. Modest-sized reflectors, of about 20 to 25 meters in diameter, have already flown in space, such as the Russian Znamya space mirror experiment [8]. Although the first space mirror experiment (Znamya 2) was a successful, the spot brightness achievable with reflectors of this size is a tiny fraction of the mid-day Sun. For example, [10] showed that the required reflector area to increase the total insolation of Mars by 30%, as part of a large-scale terraforming effort, is of order 10^{13} m^2 (and mass of order 10^{10} kg). In this manner, large-scale geo-engineering appears to be an interesting tool to explore the possibility of climate heating.

The work presented in this paper aims to investigate the feasibility of using orbiting reflectors on polar orbits in order to explore the possibility of an increase of the total

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Mars insolation to increase the mean Mars surface temperature, so that the polar caps have evaporated, delivering an atmosphere, a first step of terraforming processing. As suggested in [12], a Sun-synchronous frozen polar orbit normal to the ecliptic plane of Mars is considered. In principle, reflector orbits normal to the Sun-line are more efficient than orbits in the ecliptic plane. Polar orbits provide additional solar energy transferred to the surface of Mars. In this light, the two-body problem is considered for polar orbits, including solar radiation pressure (SRP) and the effect of the Mars's oblateness, the J_2 effect. It will be shown that SRP and the J_2 have a significant effect on the orbital evolution, obtaining analytical expressions for the required characteristic acceleration.

2 Sun-synchronous orbit

A Sun-synchronous orbit is a geocentric orbit which combines the altitude of the satellite and the inclination of the orbit, in such a way that, the speed of precession of the osculating orbital plane is approximately one degree per day with respect to the celestial sphere to keep pace with the Earth's revolution around the Sun. A frozen orbit is an orbit where the mean argument of the perigee, the eccentricity and the inclination remain constant [3]. In the two-body problem, the plane of any orbit will remain fixed with respect to an inertial frame as the Earth rotates beneath it, i.e., the orbital elements do not change with time in the inertial reference coordinates. However, the non-spherical gravitational potential perturbation, as well as the solar radiation pressure force (SRPF), could cause this plane to slowly shift. Thus, in this study J_2 and solar pressure perturbations are considered to model the dynamics of the solar reflector. The variations of the orbital elements are governed by the Gauss' form of the planetary equations (see equations (1a)-(1e)). Thus, the dynamical equations of motion of the solar reflector can be written as shown in equations (1a)-(1e) [1]

$$\frac{\mathrm{d}a}{\mathrm{d}f} = \frac{2pr^2}{\mu\left(1-e^2\right)^2}\left(T_r e\sin f + T_t\frac{p}{r}\right), \tag{1a}$$

$$\frac{\mathrm{d}e}{\mathrm{d}f} = \frac{r^2}{\mu}\left[T_r\sin f + T_t\left(1+\frac{r}{p}\right)\cos f + T_t e\frac{r}{p}\right], \tag{1b}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}f} = \frac{r^2}{\mu e}\left[-T_r\cos f + T_t\left(1+\frac{r}{p}\right)\sin f\right] - \cos i\frac{\mathrm{d}\Omega}{\mathrm{d}f}, \tag{1c}$$

$$+\frac{3J_2R^2}{2a^2\sqrt{1-e^2}}\left(2-\frac{5}{2}\sin^2 i\right)\frac{1}{\left(1+e\cos f\right)^2},$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}f} = \frac{a^2\left(1-e^2\right)^2}{\mu\sin f}\frac{\sin\left(\omega+f\right)}{\left(1+e\cos f\right)^3}T_w - \frac{3J_2R^2}{2a^2\sqrt{1-e^2}}\frac{\cos i}{\left(1+e\cos f\right)^2}, \tag{1d}$$

$$\frac{\mathrm{d}i}{\mathrm{d}f} = \frac{a^2\left(1-e^2\right)^2}{\mu}\frac{\cos\left(\omega+f\right)}{\left(1+e\cos f\right)^3}T_w, \tag{1e}$$

where a is the semi-major axis, e is the eccentricity, ω is the argument of perigee, Ω is the right ascension of the ascending node (or simply 'node'), i is the inclination, f is the

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true anomaly, $r = a\left(1-e^2\right)/\left(1+e\cos f\right)$ is the distance from the Mars, $p = a\left(1-e^2\right)$ is the semi-latus rectum, $R = 3397.2$ km is the mean radius of Mars, $J_2 = 0.001964$ is a dimensionless constant corresponding to oblateness of Mars, $\mu = 42828.37$ km^{3}s^{-2} is the gravitational parameter of Mars, and T_r, T_t, T_w are the acceleration components of the solar radiation force $\boldsymbol{T}$ along radial $\hat{\boldsymbol{r}}$, transverse $\hat{\boldsymbol{t}}$ and normal $\hat{\boldsymbol{w}} = \hat{\boldsymbol{r}} \times \hat{\boldsymbol{t}}$ directions (RTW frame), respectively, as shown in Figure 1(b).

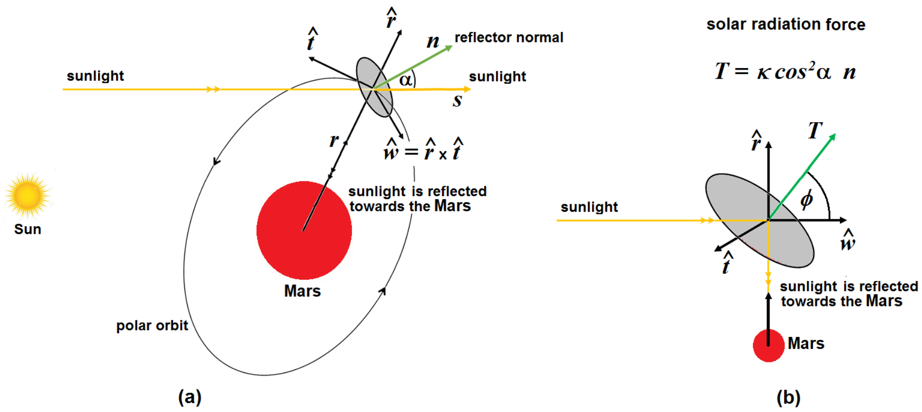

Figure 1: (a) Solar reflector on a polar orbit and radial, transverse and normal (RTN) frame, (b) Solar radiation force.

Work presented by [7] on orbital dynamics using solar sails presents analytical expressions for the SRPF components, such that, the conditions of a Sun-synchronous frozen orbit are satisfied. Firstly, setting the transverse component equal to zero (i.e. $T_t = 0$) in equations (1a)-(1e), the semi-major axis and the eccentricity remain constant over one orbital period [7]. Therefore, the normal of the reflector lies in the plane spanned by the position vector and $Z-$axis. In addition, the variation of the inclination is equal to zero over one orbital period when the argument of perigee is a multiple of $\pi/2$ (i.e. $\omega = \pm k\pi/2$) [7]. Therefore, a Sun-synchronous frozen orbit is finally achieved when the argument of perigee always remains constant over one orbital period and the node keeps pace with the Sun-line. According to [7], the radial and normal components of the SRPF that satisfy the Sun-synchronous conditions can be obtained by equations (2a)-(2b)

$$T_r = \sqrt{\frac{\mu}{a\left(1-e^2\right)}}\cos i\, n_{\odot} + \frac{3\mu J_2 R^2}{4a^4\left(1-e^2\right)^{5/2}}\left(1-5\cos^2 i\right), \tag{2a}$$

$$T_w = -\frac{\mu J_2 R^2 \sin 2i}{2\left(1-e^2\right)^{3/2} a^4 e \sin\omega} - \frac{2}{3}\sqrt{\frac{\mu\left(1-e^2\right)}{ae^2}}\frac{\sin i\, n_{\odot}}{\sin\omega}, \tag{2b}$$

where $n_{\odot} = 0.5240$ deg/day is the mean motion of the Sun. Note that equation (2b) is singular for circular orbits.

Denoting by ϕ the angle between the reflector normal and the normal component in the RTN frame (see Figure 1(b)), then the SRPF in the RTN frame is given by equations

(3)

$$T_r = T\sin\phi, \qquad T_r = 0, \qquad T_w = T\cos\phi, \tag{3}$$

where T is the magnitude of the SRPF. Thus, the angle ϕ is computed by equation (4)

$$\phi = \tan^{-1}\frac{T_r}{T_w}. \tag{4}$$

Supposing that the ecliptic plane of Mars coincides with the equatorial plane of Mars and that the space reflector is on a frozen orbit normal to the ecliptic plane of Mars, i.e. $i = 90°$, the sunlight is reflected towards Mars when $\phi = 45°$ [12], i.e. $T_r = |T_w|$. If the angle between the reflector normal and the radial vector remains constant, then equations (2a)-(2b) permit the semi-major axis of a Sun-synchronous frozen orbit, as well as the solar radiation force, to be obtained as a function of the classical orbit elements as shown in equations (5a)-(5c):

$$a = \left(-\frac{\frac{3\mu J_2 R^2}{4(1-e^2)^{5/2}}}{\frac{2}{3}\sqrt{\frac{\mu(1-e^2)}{e^2}}\frac{n_\odot}{\sin\omega}}\right)^{2/7}, \tag{5a}$$

$$T_r = \frac{3\mu J_2 R^2}{4a^4\left(1-e^2\right)^{5/2}}, \tag{5b}$$

$$T_w = -\frac{2}{3}\sqrt{\frac{\mu\left(1-e^2\right)}{ae^2}}\frac{n_\odot}{\sin\omega}. \tag{5c}$$

The Keplerian elements identified through equation (5a) are now used as initial conditions for a numerical propagation forward in time of Gauss' equations (1a)-(1e). Thus, once the initial parameters of the orbital dynamics are found, the solar acceleration components T_r, T_t, and T_w are determined to perform the numerical propagation by equations (6)

$$T_r = \kappa\cos^2\alpha\,\sin\phi, \qquad T_t = 0, \qquad T_w = \kappa\cos^2\alpha\,\cos\phi, \tag{6}$$

where $\phi = 45°$, $\kappa = \sqrt{T_r^2 + T_w^2}/\cos\alpha$ is the characteristic acceleration of the reflector [11] and the sunlight angle $\alpha = \cos^{-1}(\boldsymbol{n}\cdot\boldsymbol{s})$ (see Figure 1(a)), where $\boldsymbol{n} = [\sin\phi \;\; 0 \;\; \cos\phi]$ and $\boldsymbol{s}$ is the unit vector of sunlight direction in the RTW frame [7].

3 Results

Using equation (5a), Figure 2 shows semi-major axis a as a function of eccentricity e for frozen orbits with $i = 90°$, $\omega = 90°$. Note that we are considering quasi-circular orbits, i.e. $0 \approx e << 1$.

In this manner, two simulations were implemented as shown in Figures 3 and 4, supposing a linear relation for the position $\lambda_\odot$ of Sun and time, i.e. $\lambda_\odot = \lambda_{\odot 0} + n_\odot t$, where

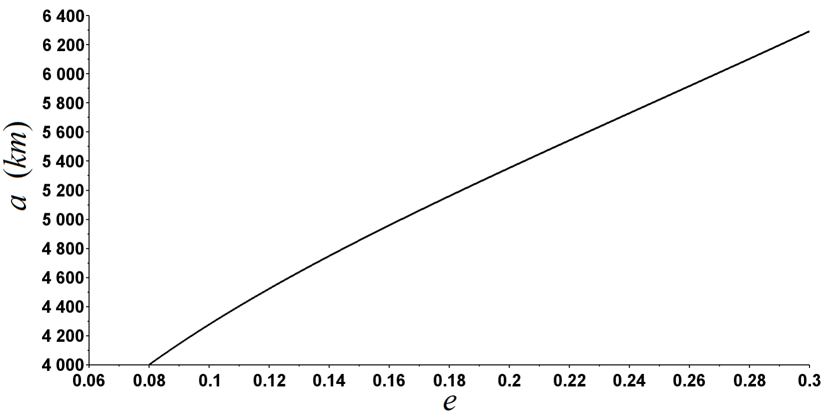

Figure 2: Semi-major axis a as a function of eccentricity e for frozen orbits with $i = 90°$, $\omega = 90°$.

$\lambda_{\odot 0}$ is the initial solar longitude at the moment of ejection ($t = 0$). Figures 3 and 4 show the variation of classical orbit elements $(a, e, \Omega, i, \omega)$ and Sun angle (α) during fifty days for two quasi-circular orbits with $e_0 = 0.08$ and $e_0 = 0.3$, respectively. As can be seen in Figure 4, when the eccentricity increases, the orbital plane keeps pace with the sunlight and the variation of the inclination i and argument of perigee ω is practically zero, achieving a frozen orbit.

Finally, it is interesting to see in Figure 4 that the right ascension of ascending node Ω follows the variation of the sunlight direction, in other words, angle α remains constant and is equal to the orientation of the solar reflector angle Φ. This fact is due to the sunlight is perpendicular to the orbit plane and it keeps pace with the sunlight.

4 Conclusions

The Gauss' form of planetary equations shows that it is possible to obtain frozen orbits for space reflectors when the J_2 and solar radiation perturbations are considered. These equations permits to obtain analytical expressions and initial conditions for the classical orbit elements and solar radiation force. The conditions for a frozen orbits could be guaranteed when the orbit eccentricity is increased, in such a way that it would be not necessary to control the orbit.

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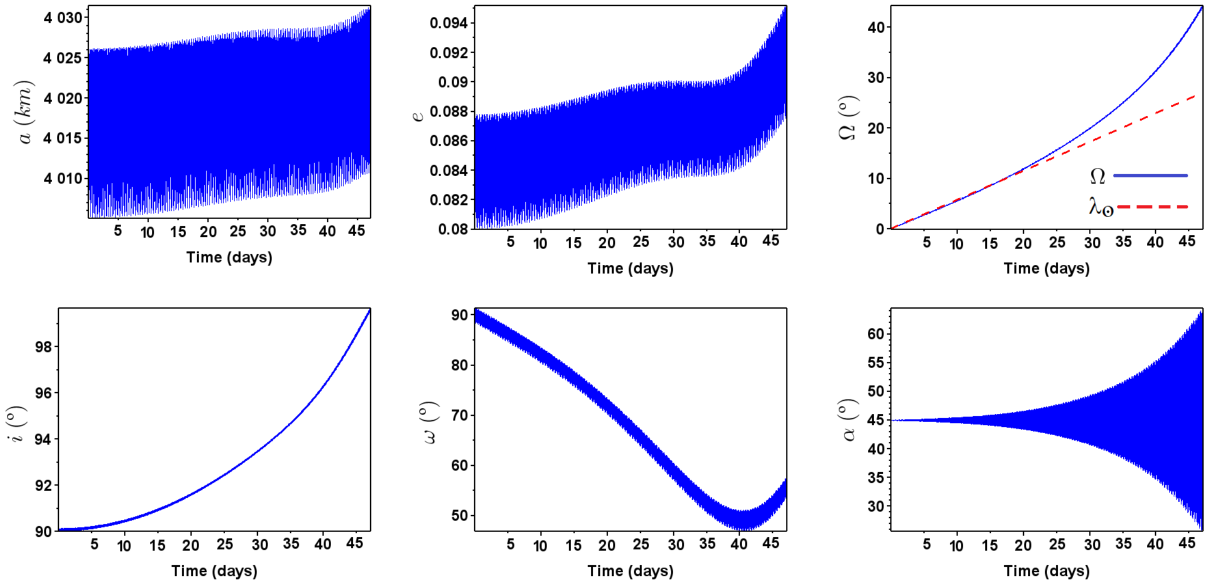

Figure 3: Classical orbit elements and Sun angle variation (α) with $e_0 = 0.08$, $i_0 = 90°$, $\omega_0 = 90°$, $\Omega_0 = 0°$, $\lambda_{\odot 0} = 3\pi/2$, and $\kappa = 8.13$ mm/s^2.

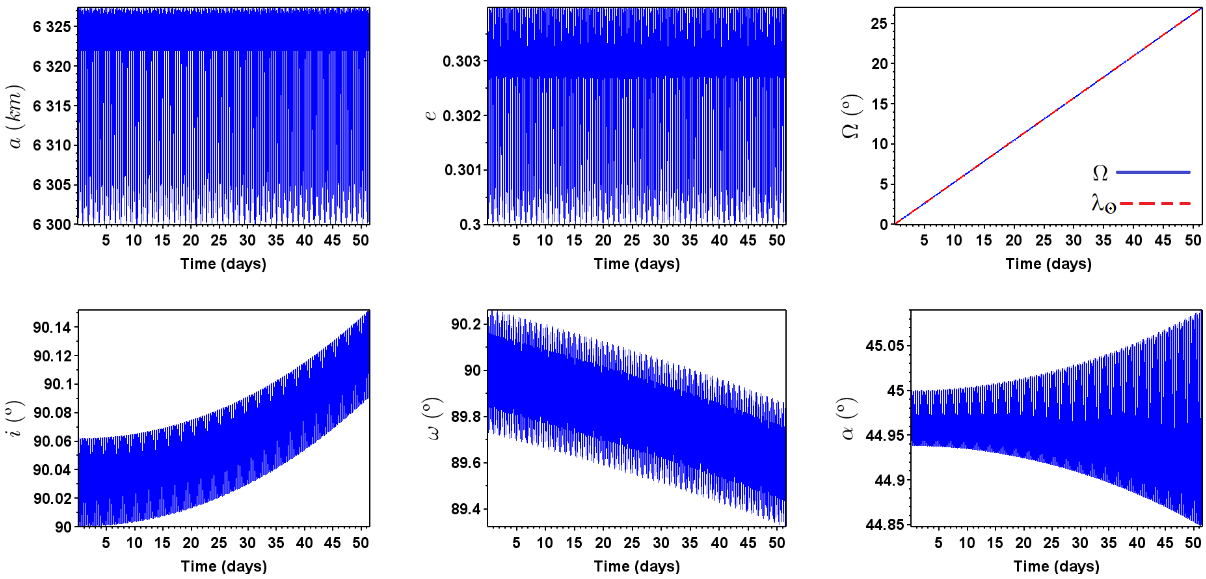

Figure 4: Classical orbit elements and Sun angle variation (α) with $e_0 = 0.3$, $i_0 = 90°$, $\omega_0 = 90°$, $\Omega_0 = 0°$, $\lambda_{\odot 0} = 3\pi/2$, and $\kappa = 1.65$ mm/s^2.

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