Proceeding Series of the Brazilian Society of Computational and Applied Mathematics

Automatic Construction of Vascular Arteriovenous Tree Geometric Model

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Abstract. We propose an algorithm based on the computational method of Constrained Constructive Optimization for construction of a geometric model of vascular arteriovenous tree. It uses physiological conditions for pressure and flow while minimizing the total intravascular volume. We apply the algorithm to construct a geometric model of renal vascular system. The results show that our model has morphometric properties similar to real renal arterial and venous trees.

Palavras-chave. Circulatory Trees, Vascular Anatomy, Geometric Models.

1 Introduction

In order to study blood circulation, we need to construct geometric models of vascular trees. There are different methods to construct these models. For example, we can use fractal methods [8, 15] or optimization methods like Constrained Constructive Optimization (CCO) [4, 11, 12]. Over the last year, new adaptations of CCO were proposed [1, 2, 7].

Given a perfusion domain, the main goal of the CCO method and its variants is to construct only one vascular tree, which could be either arterial or venous. However, in many regions of human body we have coupled arterial and venous trees.

In this work our goal is propose an algorithm based on CCO to construct two or more vascular trees, given a 3D perfusion volume not necessarily convex, that share their distal ends of vascular terminal segments. The constructed trees do not have vascular segments intersecting each other. The first description of this algorithm was given by Queiroz [9] and its potential is verified here by constructing a geometric model of renal vascular system.

2 The algorithm

Lets assume that the geometric model of arteriovenous vascular system is composed by two or more circulatory trees connected in their distal ends of vascular terminal segments.

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To construct that model we propose the Algorithm 1 based on CCO that considers each circulatory tree t ($t = 1, \ldots, N_{trees}$) in the model following the conditions:

(i) It has binary ramification of vascular segments. Each segment is represented as a rigid cylindrical tube, perfused at steady state and laminar flow conditions;

(ii) The hydrodynamic resistance R_i of each segment i is given by Poiseuille's law [3]

$$R_i = \frac{8\eta l_i}{\pi r_i^4}, \tag{1}$$

where η is the viscosity of blood (we assume constant with $\eta = 3.6$ cP), l_i and r_i are the length and internal radius;

(iii) The pressure drop Δp_i along segment i is given by

$$\Delta p_i = R_i Q_i, \tag{2}$$

where Q_i is the blood flow;

(iv) Q^t_{perf} is the blood flow along root segment of tree t, $Q^t_{term} = \frac{Q^t_{perf}}{K_{term}}$ is the blood flow along terminal segments, and K_{term} is the number of terminal segments;

(v) The total pressure drop Δp^t along tree t is given by

$$\Delta p^t = p^t_{perf} - p^t_{term}, \tag{3}$$

where p^t_{perf} is the perfusion pressure at proximal position $\mathbf{x}^t_{prox}$ of root segment of tree t, and p^t_{term} is the pressure at distal position of terminal segments;

(vi) At bifurcation the radius of parent segment (r_i) and the radii of children segments (r_{left} e r_{right}) obey the Murray's law [13]

$$r_i^\gamma = r^\gamma_{left} + r^\gamma_{right}, \tag{4}$$

where γ is assumed constant during the tree construction;

(vii) The segments are constructed in order to minimize a target function (total volume of tree) given by

$$V = \pi \sum_{i=1}^{K_{term}} l_i r_i^2. \tag{5}$$

Figure 1 illustrates the steps of Algorithm 1 used to construct a model with two vascular trees ($N_{trees} = 2$), each one with two terminal segments ($N_{term} = 2$).

Some comments about the steps of Algorithm 1:

- The value of N_{trees} can be arbitrary. For the renal vascular system $N_{trees} = 2$.

DOI: 10.5540/03.2018.006.02.0257

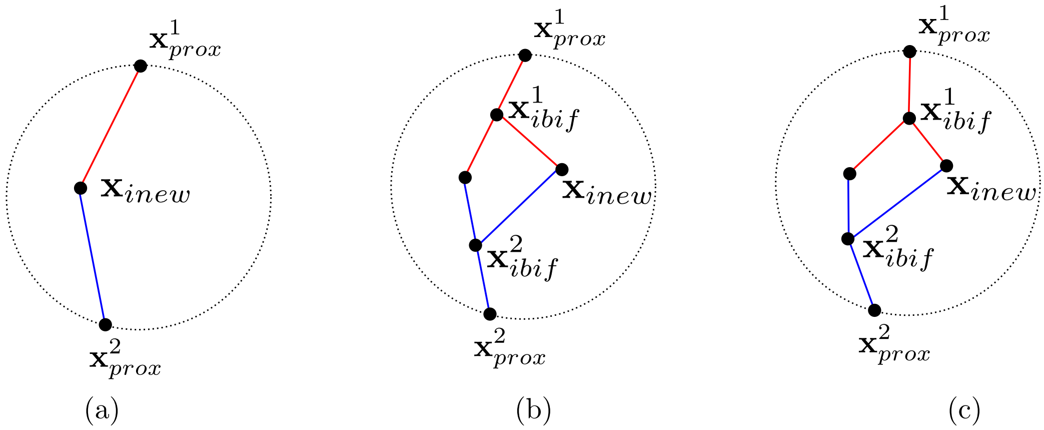

Figure 1: Construction of arterial tree (red) and venous tree (blue) models. (a) Connecting $\mathbf{x}_{inew}$ with $\mathbf{x}^1_{prox}$ and $\mathbf{x}^2_{prox}$. (b) Connecting $\mathbf{x}_{inew}$ with $\mathbf{x}^1_{ibif}$ and $\mathbf{x}^2_{ibif}$. (c) Optimizing $\mathbf{x}^1_{ibif}$ and $\mathbf{x}^2_{ibif}$.

Algorithm 1: Automatic construction of an arteriovenous vascular system.

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   Input: x^t_prox, Q^t_perf, N_term, p^t_perf, p^t_term, γ, N_trees.
1  Set the proximal position x^t_prox of the root segments in perfusion domain;
2  Generate and validate the root segments terminal position x_inew;
3  for t ← 1 to N_trees do
4      Connect x_inew to x^t_prox (Add root segment in tree t);
5  K_term = 1;
6  while ( K_term < N_term)
7      Generate and validate new terminal segment distal position x_inew;
8      Connect x_inew to a segment in tree t creating new bifurcation position x^t_ibif;
9      Adjust x^t_ibif in order to minimize the target function;
10     K_term = K_term + 1;
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- At lines 2 and 7 the generated position $\mathbf{x}_{inew}$ is valid if it follows a distance criterion [9, 11] considering all segments of tree t (otherwise another position is generated);

- At line 8 when we connect $\mathbf{x}_{inew}$ to a segment in tree t, we create a new bifurcation position $\mathbf{x}^t_{ibif}$. This position has to be adjusted in order to minimize the target function (5). The detailed description of this minimization process is presented in [9].

3 Results

In this section we discuss the Algorithm 1 applied to construct a geometric model of the renal arteriovenous system, which is formed by two trees.

The parameters used in our implementation are presented in Table 1 and they can be seen in [5]. The proximal position of the root segment for each tree was determined from the vascular network model in Anatomium^{TM} [6] and the main branches of the tree presented in [5]. The perfusion domain surface representing the kidneys was also determined from [6].

Table 2 shows the morphometric data for the model trees. The value n_{max} shows the maximum bifurcation, i.e., the maximum number of proximal bifurcations along the path from the respective segment to the root segment. We denote the arterial tree as AT and the venous tree as VT. The mean and standard deviation were calculated from 10 generated models. It is important to notice that the root segment radius (r_{iroot}) for AT and VT are consistent with real renal vascular trees. As we can see in [10, 14], the real renal arterial tree has a root segment radius from 2 mm to 6 mm and the real renal venous tree has a root segment radius from 5 mm to 7 mm. It is also consistent with real renal vascular trees whose the root segment radius and the intravascular volume (V) for VT are greater than those values for AT.

An example of renal vascular tree model generated by Algorithm 1 is illustrated in Figure 2. The arterial tree (red) and venous tree (blue) were separated for better visualization. This model is consistent with real renal vascular system, whose vascularization is concentrated at the kidney's parenchyma, i.e. the functional parts of this organ.

In Figure 3 we show the morphometric curves relating the mean segment diameter and the range of diameters (i.e. their S.D.) with bifurcation level. These curves have a decay behavior like in experimental data from vascular corrosion casting [16].

Table 1: Parameters used in the renal vascular tree model.

Parameter	Arterial tree	Venous tree
p^t_{perf} (mmHg)	95	10
p^t_{term} (mmHg)	15	5
Q^t_{perf} (mL/min)	617.5	617.5
N_{term}	3200	3200
γ	2.2	2.2

Table 2: Morphometric data for the renal vascular tree model.

Tree	r_{iroot} (mm)	V (mm^3)	$n_{\max}$
AT	2.0900 ± 0.0019	720.0206 ± 6.6782	44 ± 2
VT	4.0862 ± 0.0106	2797.5518 ± 23.1052	48 ± 4

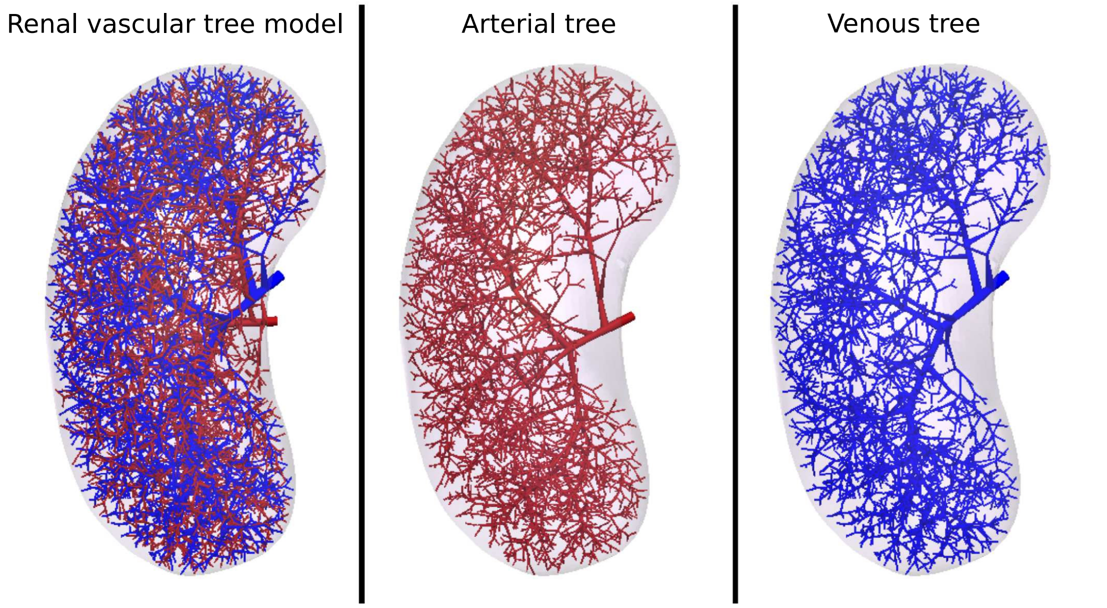
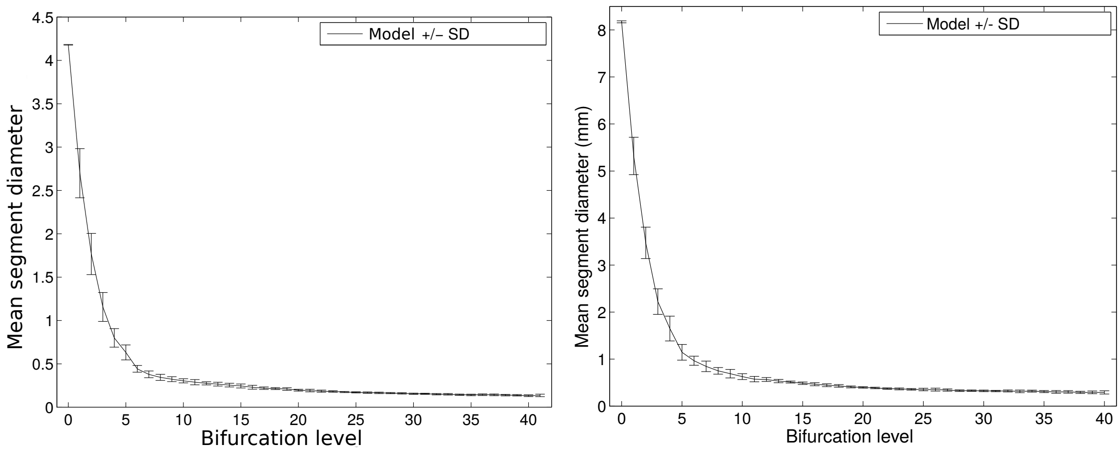


Figure 2: Renal vascular tree model. (Adapted from [9].)



(a) (b)

Figure 3: Morphometric curves relating the mean segment diameter with bifurcation level. (Adapted from [9].) (a) Arterial tree. (b) Venous tree.

4 Conclusion and future work

In this work we verify the potential of an algorithm based on CCO to construct a geometric model of vascular arteriovenous system. We applied this algorithm to construct a renal vascular tree model with two circulatory trees. The model has morphometric properties consistent with real renal vascular tree.

DOI: 10.5540/03.2018.006.02.0257

As future work we will intend to apply the proposed algorithm to construct a liver vascular tree model that is composed by more than two circulatory trees.

Acknowledgment

The authors would like to acknowledge the financial support provided by the Brazilian agencies FAPEMIG (Proc. Num. 00795-14) and CNPq (through the *Instituto Nacional de Ciência e Tecnologia em Medicina Assistida por Computação Científica* – INCT-MACC).

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DOI: 10.5540/03.2018.006.02.0257