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Stochastic model for Tomas Hobbes's Morality

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Abstract. In this work we present an one-dimensional lattice stochastic model inspired in the moral theory of philosopher Thomas Hobbes [7]. The lattice is formed by L sites which can be occupied by a individual (moral or immoral) and a fiscal. Through of Monte Carlo Simulation we observed a second order phase transition from active to non-unique absorbing state.

Keys-word. Stochastic Process, Non-Equilibrium Phase Transitions, Monte Carlo Simulation

1 Introduction

The ethic is a branch of knowledge which started formally with the development of Greek philosophy [3,11] which remain the subject of studies by contemporary philosophers [5, 12, 13]. A master key issue has been taken in line with moral and immoral attitudes within a society.

Thomas Hobbes [7] was the first modern philosopher that offered naturalist principle to ethic. In his theory, the ethic appears when people understand the necessary conditions to live well.

According to Hobbes, these conditions are defined by imposition of equality of rights, by necessity of auto preservation and by fixation of deals among individuals. Hereby, a deal is defined as mutual exchange of rights. In this way, who makes a deal is classified as fair or moral person, otherwise, as unfair or immoral person.

In this philosophy the level of morality of society is controlled by the action of one or more supervisory agents, i.e., fiscals. These, in turn, promote measures to curb actions classified as immoral, in exchange for the possession of a kind of "absolute power". In reality the idea of an absolute power can be translated as a capacity that the rulers holds to influence the decisions of the individuals of the society.

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Currently in many cases of corruption around the world, we realized the collapse of the concept of morality inside the society was lost due to the influence of some immoral agents in the system.

Inspired by this ideas, we present a stochastic model with asynchronous dynamics to observe the colletive effects produced by tax pressure on the individuals of a society. In particular, we will be interested in determining the level of morality, the possible asymptotic collective states and their respective non equilibrium phase transition.

This model exhibits a continuous phase transition from an active state to an inactive (absorbing state) [9, 10]. The active state is formed by a statistical mixture of moral, immoral, and fiscal individuals. The inactive (absorbent) state is defined when all the individuals in the lattice are in the moral state, regardless of the number of fiscals initially set in the sites. That is, for each fiscal density there is a different absorbing state.

Differently from the stochastic lattice approach, in current literature there are several works that deal with the taxe evasion problem (moral economic problem) by equilibrium statistical mechanics [4, 14, 15]. However, the most common analysis about the morality dynamics has been formulated in context of evolutionary game theory [1, 2].

2 The Model

The model is defined in a one-dimensional latice with L sites and periodic boundary conditions. At each site is permited to have, simultaneously, two different states, i.e; one for the individual and another for the fiscal. The general configuration $\vec{\mathfrak{C}}$ of the system (society) is defined by

$$\vec{\mathfrak{C}} = (\mathfrak{C}_1; \mathfrak{C}_2; \ldots; \mathfrak{C}_L) \tag{1}$$

where $\mathfrak{C}_i = (\mathrm{I}_i, \mathrm{F}_i)$. The variable I_i assume the value 1 when the individual at the site i is in the moral state, or the value 0 when the individual is in the immoral state. Besides that, If at site i there is a fiscal we write $\mathrm{F}_i = \mathrm{Y}$ (YES), otherwise $\mathrm{F}_i = \mathrm{N}$ (NO).

The dynamic site i is not directly influenced by the presence of fiscals in the neighborhood $i-1$ and $i+1$. The evolution of the rules of the model are as follows: A site i is chosen randomly among L sites of the lattice At each time step ($\Delta t = 1/L$) a site i is chosen randomly among L sites of the lattice.

If we have a fiscal at the sitie i and for any time t , i.e; $\mathrm{F}_i \equiv \mathrm{Y}$, then the transitions rate ω of state $\mathrm{I}_i = l_1$ goes to state $\mathrm{I}_i = l_2$ at the time $t + \Delta t$ given that the states $\mathrm{I}_{i-1} = j$ and $\mathrm{I}_{j+1} = m$ are remained unchanged is

$$P(\mathrm{I}_i = l_2, t + \Delta t | \mathrm{I}_{i-1} = j, \mathrm{I}_i = l_1, \mathrm{I}_{i+1} = m, \mathrm{F}_i = \mathrm{Y}, t) := \mathbb{W}(\mathrm{Y})^{j\, l_1\, m}_{j\, l_2\, m} \tag{2}$$

where

$$\mathbb{W}(\mathrm{Y})_{j\ l_2\ m}^{j\ l_1\ m} = \begin{cases} \delta_{l_2,1}\delta_{j,1} + \delta_{l_1,1}\delta_{j,0}[r\delta_{l_2,1} + (1-r)\delta_{l_2,0}] + \\ +q\delta_{l_1,0}\delta_{l_2,1}\delta_{j,0} + (1-q)\delta_{l_1,0}\delta_{l_2,0}\delta_{j,0} & \text{if} \quad j = m \\ \\ r_1\delta_{l_1,1}\delta_{l_2,0} + (1-r_1)\delta_{l_1,1}\delta_{l_2,1} + \\ +r_0\delta_{l_1,0}\delta_{l_2,1} + (1-r_0)\delta_{l_1,0}\delta_{l_2,0} & \text{if} \quad j \neq m \end{cases}$$

and

$$P(I_i = l_2, t+\Delta t | I_{i-1} = j, I_i = l_1, I_{i+1} = m, F_i = \mathrm{N}, t) := \mathbb{W}(\mathrm{N})_{j\ l_2\ m}^{j\ l_1\ m} \tag{3}$$

$$\mathbb{W}(\mathrm{N})_{j\ l_2\ m}^{j\ l_1\ m} = \begin{cases} \delta_{j,l_2} & \text{if} \quad j = m \\ \\ w_1\delta_{l_1,1}\delta_{l_2,0} + w_0\delta_{l_1,0}\delta_{l_2,1} + \\ +(1-w_1)\delta_{l_1,1}\delta_{l_2,1} + (1-w_0)\delta_{l_1,0}\delta_{l_2,0} & \text{if} \quad j \neq m \end{cases}$$

Explicitary, the probability transition for situation with NO fiscal at the sitie i

$$\begin{gathered} \mathbb{W}(\mathrm{N})_{1\,1\,1}^{1\,1\,1} = \mathbb{W}(\mathrm{N})_{1\,1\,1}^{1\,0\,1} = \mathbb{W}(\mathrm{N})_{0\,0\,0}^{0\,1\,0} = \mathbb{W}(\mathrm{N})_{0\,0\,0}^{0\,0\,0} = 1, \\ \\ \mathbb{W}(\mathrm{N})_{1\,0\,1}^{1\,1\,1} = \mathbb{W}(\mathrm{N})_{1\,0\,1}^{1\,0\,1} = \mathbb{W}(\mathrm{N})_{0\,1\,0}^{0\,1\,0} = \mathbb{W}(\mathrm{N})_{0\,1\,0}^{0\,0\,0} = 0, \end{gathered} \tag{4}$$

$$\begin{gathered} \mathbb{W}(\mathrm{N})_{0\,0\,1}^{0\,1\,1} = \mathbb{W}(\mathrm{N})_{1\,0\,0}^{1\,1\,0} = w_1 \quad , \quad \mathbb{W}(\mathrm{N})_{0\,1\,1}^{0\,1\,1} = \mathbb{W}(\mathrm{N})_{1\,1\,0}^{1\,1\,0} = 1 - w_1, \\ \\ \mathbb{W}(\mathrm{N})_{0\,1\,1}^{0\,0\,1} = \mathbb{W}(\mathrm{N})_{1\,1\,0}^{1\,0\,0} = w_0 \quad , \quad \mathbb{W}(\mathrm{N})_{0\,0\,1}^{0\,0\,1} = \mathbb{W}(\mathrm{N})_{1\,0\,0}^{1\,0\,0} = 1 - w_0, \end{gathered} \tag{5}$$

and when we have ONE fiscal at the sitie i

$$\begin{array}{ccc} \mathbb{W}(\mathrm{Y})_{1\,1\,1}^{1\,1\,1} & = & \mathbb{W}(\mathrm{Y})_{1\,1\,1}^{1\,0\,1} = 1, \\ \mathbb{W}(\mathrm{Y})_{0\,1\,0}^{0\,1\,0} = r & , & \mathbb{W}(\mathrm{Y})_{0\,0\,0}^{0\,1\,0} = 1 - r, \\ \mathbb{W}(\mathrm{Y})_{0\,1\,0}^{0\,0\,0} = q & , & \mathbb{W}(\mathrm{Y})_{0\,1\,0}^{0\,0\,0} = 1 - q, \end{array} \tag{6}$$

$$\begin{gathered} \mathbb{W}(\mathrm{Y})_{0\,1\,1}^{0\,0\,1} = \mathbb{W}(\mathrm{Y})_{1\,1\,0}^{1\,0\,0} = r_0 \quad , \quad \mathbb{W}(\mathrm{Y})_{0\,0\,1}^{0\,0\,1} = \mathbb{W}(\mathrm{Y})_{1\,0\,0}^{1\,0\,0} = 1 - r_0, \\ \\ \mathbb{W}(\mathrm{Y})_{0\,0\,1}^{0\,1\,1} = \mathbb{W}(\mathrm{Y})_{1\,0\,0}^{1\,1\,0} = r_1 \quad , \quad \mathbb{W}(\mathrm{Y})_{0\,1\,1}^{0\,1\,1} = \mathbb{W}(\mathrm{Y})_{1\,1\,0}^{1\,1\,0} = 1 - r_1, \end{gathered} \tag{7}$$

Parameterization

Without loss of generality, let us fixe the parameters q, r_0, r_1, w_0 and w_1 in terms of r ("mean efficiency of fiscalization") and of difference Δ between the "free will probability " for don't fulfill a contract w_1, and the "free will probability " for fulfill a contract w_0. Chosen $w_1 = 1 - w_0$ we may write

$$\begin{aligned} q &= r \qquad (8) \\ \Delta = w_0 - w_1 &= 1 - 2w_0. \qquad (9) \end{aligned}$$

If $\Delta < 0$ the society behaves, in average, "honestly", on another hand when $\Delta > 0$ $(0 < w_0 \leq \frac{1}{2})$ the society behaves "corruptly". We are interested to set up (number of fiscals, level of fiscalization) to society with corruption $0 < \Delta \leq 1$ reaches the highest level of morality possible.

The probabilities r_0 and r_1 should be parameterized so that when the fiscalization efficiency is null ($r = 0$) we have $r_0 = w_0$ e $r_1 = w_1$, and when $r = 1$ necessarily we should have $r_0 = 1$ e $r_1 = 0$. The simplest way is through the linear parameterization

$$r_0 = r + (1 - r)(\frac{1 - \Delta}{2}), \qquad (10)$$

$$r_1 = (1 - r)(\frac{1 + \Delta}{2}), \qquad (11)$$

on the other words, the model will only have three free parameters,

$$0 \leq r \leq 1, ; \quad 0 \leq \Delta \leq 1, ; \quad \text{and} \quad 0 \leq \rho_F \leq 1. \qquad (12)$$

3 Monte Carlo Simulation

Any observable $\langle \Theta(t) \rangle$ of the model is calculated by performing means on the different positional configurations of moral and immoral individuals for a number fixed of fiscals (along the lattice) from the initial instant until the instant t. In the absence of fiscal ($\rho_F = 0$) the system presents two absorbing states:

(i) For any initial density of moral individuals ($\rho_I(t = 0)$) e $\Delta = w_1 - w_0 > 0$, the sites of the lattice reach the steady density $\rho_{IS} = \rho_I(t \to \infty) = 0$;

(ii) When $\Delta < 0$ e $(\rho_I(t = 0)) \neq 0$ the system reaches the state density $\rho_{IS} = 1$;

(iii) When $\Delta = 0$ the stationary density depends of initial conditions. If $\rho_I(0) > \frac{1}{2}$ we have $\rho_{IS} = 1$, otherwise $\rho_{IS} = 0$.

When we have fiscals in lattice ($\rho_F \neq 0$) the situation is more complex. To understand this we shall introduce the level of morality in the society (lattice) by $M(t, \Delta, \rho_F, r) \equiv \rho_I(t, \Delta, \rho_F, r)$. We get the stationary state through

$$M_s(\Delta, \rho_F, r) \equiv \langle \lim_{t\to\infty} \rho_I(t, \Delta, \rho_F, r)\rangle, \tag{13}$$

where the $\langle\cdot\rangle$ is valued for different initial spatial configurations of individuals and fiscals(initially we set a uniform distribution of fiscals in the Monte Carlo Simulation).

Due the conservation of morals and immoral individuals for any time, we can define the immorality of the system

$$I(t, \Delta, \rho_F, r) = 1 - M(t, \Delta, \rho_F, r). \tag{14}$$

The simulation was repeated a number of times, of the order of a ten thousand, and the averages of relevant quantities were obtained when the flutuation around the mean value was smaller than 10^{-3}. This model presents a complexity phase diagram and its complete classification in the universality class involves the knowledge of all static and dynamic critical exponents [6]. However, in this first work, we will show only the behavior of stationary immorality as function of ρ_F at different values of r and Δ.

The continuous phase transition, from the inactive state ($I_s \neq 0$) to absorbing one ($I_s = 0$), take place at the specifics points r_c which depend of ρ and Δ. In the figure 1 we plotted the stationary immorality $I_s(\Delta = 0.5)$ as function of r for two different values of $\rho_F = 0.6$ and $\rho_F = 0.9$. Notice that higher values of ρ_F implies higher values of r_c.

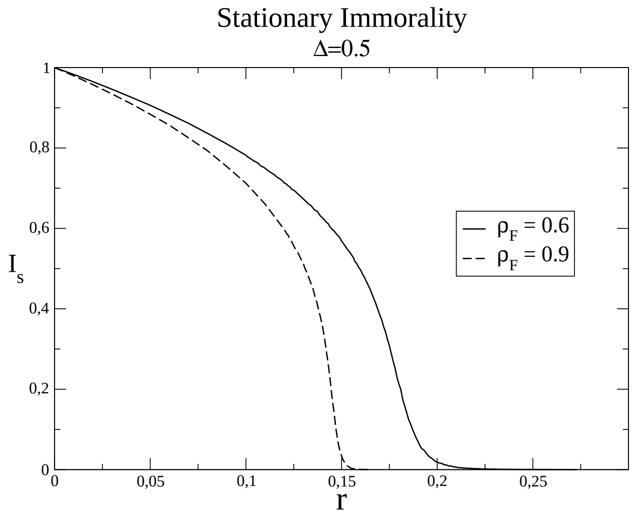

Figure 1: The Stationary Immorality $I_s(\Delta = 0.5)$ as a function of the parameter r for different values of $\rho_F = 0.6$ and $\rho_F = 0.9$.

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4 Conclusion

In the present work we introduce an one-dimensional stochastic model to investigate quantitatively the consequences of Thomas Hobbes's moral theory. In this theory society is formed by a centralizing (absolute) government that controls social relations among individuals. These in turn are classified as moral (when they fulfill the contracts established between the individuals of the society) and immoral (when they do not fulfill such contracts).

In our model the individuals are arranged in a one-dimensional lattice (with periodic boundary conditions) formed by L sites. In each site there is a moral individual (represented by the number 1) or an immoral individual (represented by 0), or else, a fiscal (represented by the number 2) that influences the rules of moral evolution of individuals.

Through Monte Carlo simulations we observe that in the absence of ($\rho_F = 0$) the model has two absorbing states. One of these states is formed when all individuals are moral ($\rho_1 = 1$) and the other when all individuals are immoral ($\rho_1 = 1 - \rho_0 = 0$).

In the presence of fiscals ($\rho_F \neq 0$) the model presents a second-order non equilibrium phase transition between the active state (statistical mixture of moral, immoral and fiscal individuals) and the absorbing state moral and fiscal). For each density of fiscal there is a different state absorbing, and thus, when $L \to \infty$ the system have infinite absorbing states.

In other words, defining the morality of society (lattice) by the fraction of moral individuals (sites in the state 1), namely, $M(t) = \rho_1(t)$, we observe that at the critical point the stationary morality reaches its maximum value ($M_S = 1$) or the minimum of imorality $I_S = 1 - M_S = 0$.

Now, we have been working to calculate the static and dynamical critical exponents, and after that, we will simulate this model in the complexy network.

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