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Pitfalls in the dynamics of coupled electromechanical systems

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Abstract. One of the main features of electromechanical systems is the mutual influence between electrical and mechanical parts. This interaction characterizes coupling. Each part of the system affects the behavior of the other. To properly represent the dynamics of a coupled system, it is necessary to properly characterize how is this interaction between the parts. Any change in model of the interaction affects the behavior of the entire system. Typically, the coupling between electrical and mechanical parts is expressed by a set of coupled differential equations. The dynamics of the coupled system is given by an initial value problem comprising this set of coupled differential equations. Some references in the literature claim that it is possible to reduce the number of equations in initial value problem without changing the interaction between the electrical and mechanical parts. They assume a hypothesis that a term in the equations can be neglected in a way that the coupling between the parts becomes a linear algebraic relationship. This hypothesis reduces the number of equations to be integrated, however it is a pitfall! It implies the decoupling of the motor-cart system, misleading the results as it is shown in this paper.

Keywords. Electromechanical system, Parametric excitation, Coupled systems

1 Introduction

Coupled systems present an interesting behavior, usually nonlinear, characterized by the mutual influence between the parts of the system [5,6]. Each part of the system affects the behavior of the other in a way that the coupling between them varies with the coupling condition.

In this paper, we are interested in a specific type of coupling: electromechanical. We analyze systems with an electrical and a mechanical parts. Typically, in this kind of system, there is a geometrical constraint between the electrical and mechanical parts. To illustrate, we present two simple electromechanical systems composed by a cart coupled to a DC motor, see Figs. 1(a) and 1(b). The difference between these two systems is the mechanism that couples the electrical and mechanical parts. In Fig. 1(a), it is shown a mechanism called scotch yoke and in Fig. 1(b), slider crank mechanism. Both of them relate the horizontal cart motion x with the motor rotational motion α, i.e., introduces a constraint between these two variables.

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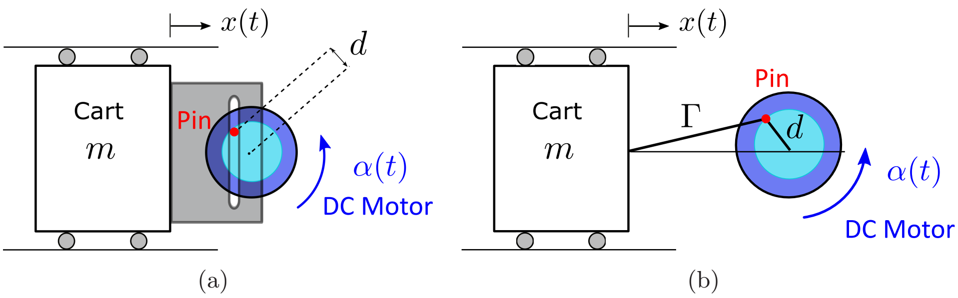

Figure 1: (a) Electromechanical system with a scotch yoke mechanism. (b) Electromechanical system with a slider crank mechanism.

In this work, we focus on the system with the scotch yoke mechanism. To translate the results to the other case is trivial. Due to the system geometry, $x(t)$ and $\alpha(t)$ are related by the following constraint

$$x(t) = d\,\cos\left(\alpha(t)\right). \tag{1}$$

2 Dynamics of an Electromechanical System

To determine the dynamics of the electromechanical system sketched in Fig. 1(a), first we derive the equations of the dynamics of each part of systems, electrical (DC motor) and mechanical (cart). After we couple the equations by the coupling torque that exists between the parts and the geometric constraint given by Eq. (1).

The mathematical modeling of DC motors is based on the Kirchhoff's law. It is written as

$$l\dot{c}(t) + r\;c(t) + k_e\dot{\alpha}(t) = \nu\;, \tag{2}$$

$$j_m\ddot{\alpha}(t) + b_m\dot{\alpha}(t) - k_e\;c(t) = -\tau(t)\;, \tag{3}$$

where t is the time, ν is the source voltage, c is the electric current, $\dot{\alpha}$ is the angular speed of the motor, l is the electric inductance, j_m is the inertia moment of the motor, b_m is the damping ratio in the transmission of the torque generated by the motor to drive the coupled mechanical system, k_e is the motor electromagnetic force constant and r is the electrical resistance. The available torque delivered to the coupled mechanical system is represented by τ, that is the component of the torque vector $\boldsymbol{\tau}$. Assuming that τ and ν are constants in time, the motor reaches a steady state in which the electric current and the angular speed become constants in time. When τ is not constant in time, the angular speed of the motor shaft and the current do not reach a constant value. This kind of situation happens when, for example, a mechanical system is coupled to the motor. In this case, $\dot{\alpha}$ and c variate in time in a way that the dynamics of the motor will be influenced by the coupled mechanical system. To model the coupling between the motor and the mechanical system, the motor shaft is assumed to be rigid. Thus, the available torque vector to the coupled mechanical system, $\boldsymbol{\tau}$, can be written as

$$\boldsymbol{\tau}(t) = \boldsymbol{d}(t) \times \mathbf{f}(t)\,, \tag{4}$$

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where $\boldsymbol{d} = (d\ \cos\alpha(t),\ d\ \sin\alpha(t),\ 0)$ is the vector related to the eccentricity of the pin, and where $\mathbf{f}$ is the coupling force between the DC motor and the cart. Assuming that there is no friction between the pin and the slot, the vector $\mathbf{f}$ only has a horizontal component, f (the horizontal force that the DC motor exerts in the cart). The available torque τ is written as

$$\tau(t) = -f(t)\ d\ \sin\alpha(t)\,. \tag{5}$$

Due to constraints, the cart is not allowed to move in the vertical direction. The cart mass is m and the horizontal cart displacement is represented by x. Since the cart is modeled as a particle, it satisfies the equation

$$m\ \ddot{x} = f(t)\ . \tag{6}$$

Substituting Eqs. (4) to (1) into Eqs. (2) and (3), we obtain the initial value problem for the motor-cart system that is written as follows. Given a source voltage ν, find $(\alpha,\, c)$ such that, for all $t > 0$,

$$l\ \dot{c}(t) + r\ c(t) + k_e\ \dot{\alpha}(t) = \nu\ , \tag{7}$$

$$\ddot{\alpha}(t)\ \left[j_m + md^2(\sin\alpha(t))^2\right] + \dot{\alpha}\ \left[b_m + md^2\dot{\alpha}(t)\cos\alpha(t)\ \sin\alpha(t)\right] - k_e\ c(t) = 0\ , \tag{8}$$

with the initial conditions

$$\dot{\alpha}(0) = 0 \quad , \quad \alpha(0) = 0 \quad , \quad c(0) = \frac{\nu}{r}\,. \tag{9}$$

Observe that the dynamics of the coupled system is given by an initial value problem comprising a set of coupled differential equations. There are some references in the literature, as the recent article published by Avanço R.H. (see [2]) and [1, 3, 4], that claim that the inductance of the armature can be neglected due to the fact that the electrical time constant of the motor l/r is usually much smaller than the mechanical time constant $rj_m/{k_e}^2$. Please remark, that this is a hypothesis based only on parameters values, it does not depend on the system being studied. In this paper we show that this hypothesis is far from true, misleading the results. To exemplify how it misleads, we perform simulations neglecting the inductance and not neglecting it and comparing the two results. One sees, immediately, the big difference between the two dynamics. The system simulated is the motor-cart system with the scotch yoke mechanism. The hypothesis implies the decoupling of the motor-cart system, this is a pitfall. The simplification of the calculations modifies the dynamics!

3 Results of Numerical Simulations

For computation, the initial value problem defined by Eqs. (7) to (9) was integrated in a range of $[0.0, \quad 6.0]$ seconds. The 4th-order Runge-Kutta method is used for the time integration scheme with a time-step equal to 10^{-6}. The motor parameters used in all simulations are listed in Table 1. The cart mass is 5.0 kg. Observe that, with these values, one has $\frac{l}{r} = 6.12\times10^{-4}$ and $\frac{rj_m}{{k_e}^2} = 1.31\times10^{-2}$. To demonstrate how the neglect of the inductance misleads the results, we perform simulations neglecting and not neglecting

Parameter	**Value**
l	1.880×10^{-4} H
j_m	1.210×10^{-4} Kg m^2
b_m	1.545×10^{-4} Nm/(rad/s)
r	$0.307\ \Omega$
k_e	5.330×10^{-2} V/(rad/s)

Table 1: Values of the motor parameters used in simulations.

it and we compare the results. The simulations were computed for different values of ν, the source voltage, and of d, the pin eccentricity. Figures 2(a) and 2(b) show the phase portrait of $\dot{\alpha}$ graph as function of c for different values of ν. In these simulations $d = 0.05$ [m]. Observe that the results are different. When the inductance is neglected, there is a linear relation between $\dot{\alpha}$ and c. This can be verified by Eq. (7). If $l\,\dot{c}(t) = 0$, it is possible to write:

$$c(t) = \frac{\nu}{r} - \frac{k_e}{r}\,\dot{\alpha}(t). \tag{10}$$

Considering this, the current is no more a variable of the system. The initial value problem is reduced to only one equation (Eq. (8)) with only two initial conditions ($\dot{\alpha}(0)$ and $\alpha(0)$). The initial value of the current is established by Eq. (10), i.e., $c(0)$ is related with $\dot{\alpha}(0)$.

Observe that when the inductance is not neglected, there is no functional relation between $\dot{\alpha}$ and c. The relation depends on initial conditions.

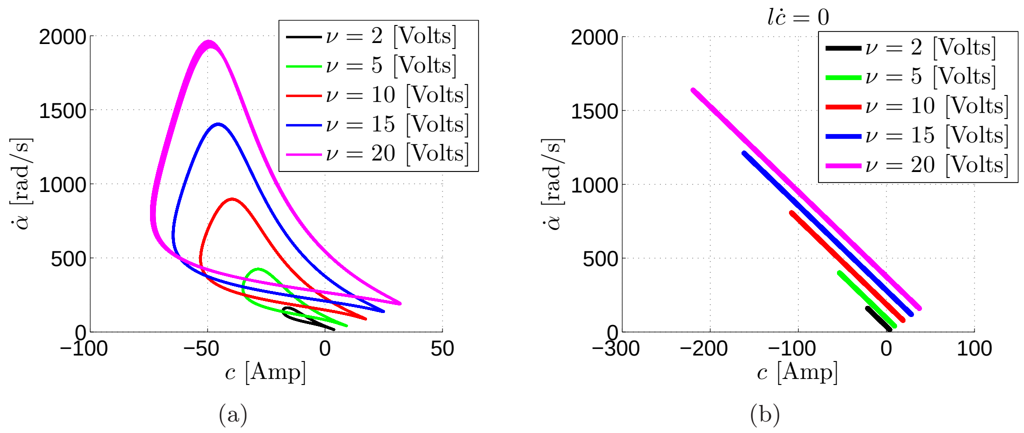
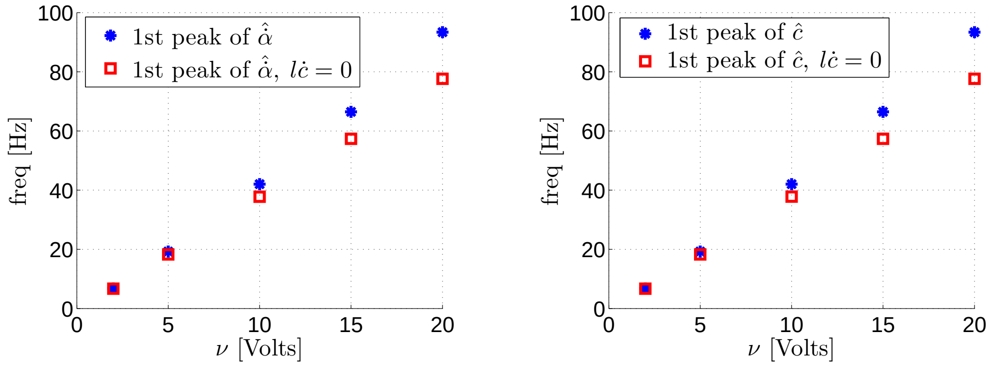


Figure 2: Phase portrait of the (a) not neglecting the inductance and (b) neglecting it, i.e., considering $l\dot{c} = 0$.

To quantify how the neglect of the inductance misleads the results, and also to enrich the analysis, we computed the Fast Fourier Transform (FFT) of the current and motor speed over time, $\hat{c}$ and $\hat{\dot{\alpha}}$. This tool have been used in the analysis of electromechanical systems (please see [7,8]). It provides important information of the signals in the frequency domain. The FFT was computed for the cases in which the inductance is neglected and is not neglected. Figures 3(a) and 3(b) show the value of frequency which correspond to the first peak of the FFT of the current and motor speed for different values of ν. In these simulations $d = 0.05$ m. Observe that as the value of ν grows, the difference between the frequencies of the first peak neglecting and not neglecting the inductance also grows.



(a) (b)

Figure 3: Values of the first peak (not neglecting and neglecting the inductance) for the FFT of (a) current (b) motor speed.

In [8] it is discussed the influence of the nominal eccentricity of the pin, the parameter d, in the dynamics of the system. It is shown that this parameter is related with the nonlinearity of the system. When d is small, the initial value problem of the motor-cart system tends to a linear system, But as the eccentricity grows, the nonlinearities become more pronounced.

To show how the neglect of the inductance misleads the results, specially when the nonlinearities are more pronounced, we perform simulations neglecting and not neglecting it and we compare the results for different values of d. Figures 4(a) and 4(b) show the phase portrait of $\dot{\alpha}$ graph as function of c for different values of d. In these simulations $\nu = 20.0$ [V]. Figures 5(a) and 5(b) show the value of frequency which correspond to the first peak of the FFT of the current and motor speed for different values of ν. In these simulations $\nu = 20.0$ Volts. Observe that as the value of ν grows, the difference between the frequencies of the first peak neglecting and not neglecting the inductance also grows.

4 Conclusions

In this paper, a simple electromechanical system was analyzed. It is shown that the inductance of the armature must not be neglected in the dynamics. Neglecting it implies the decoupling of the motor and cart! As was shown, the neglect introduces in the system a linear algebraic relationship between $\dot{\alpha}$ and c. This misleads the results, since there is no functional relation between these two variables. The lack of a functional relation is the essence of the coupling! Coupling means the equations cannot be dealt separately.

The dynamics of the coupled system is given by an initial value problem comprising a set of coupled differential equations. Any change in this set of coupled differential

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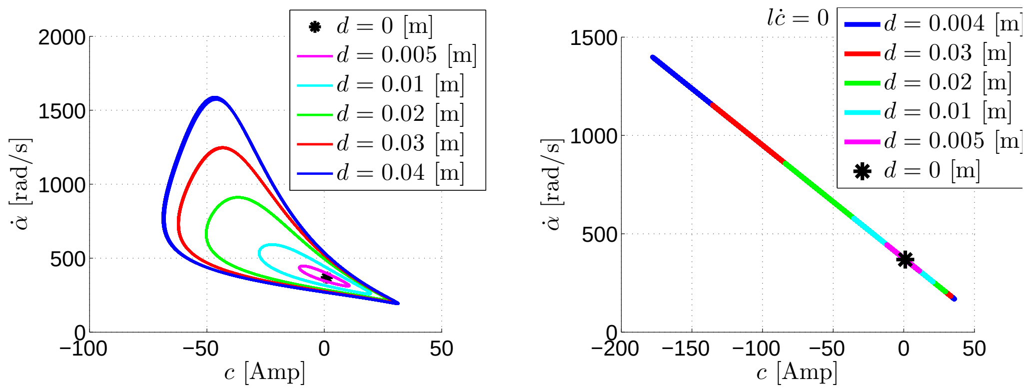
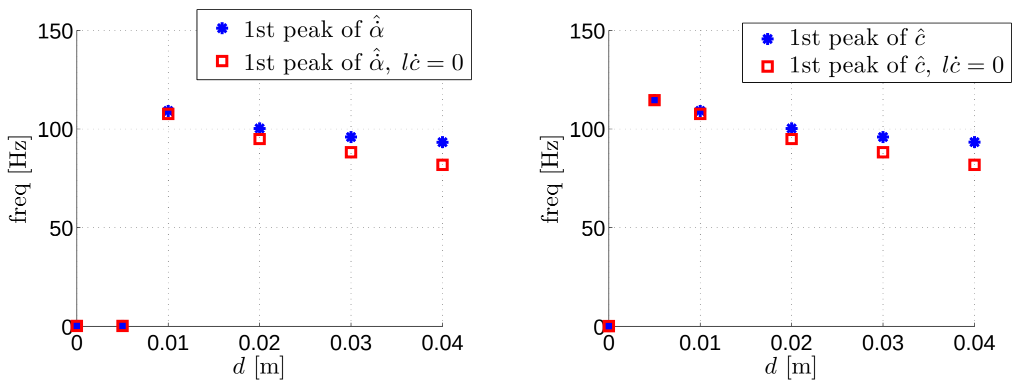


(a) (b)

Figure 4: Phase portrait of the (a) not neglecting the inductance and (b) neglecting it, i.e., considering $l\dot{c} = 0$.



(a) (b)

Figure 5: Values of the first peak (not neglecting and neglecting the inductance) for the FFT of (a) current (b) motor speed.

equations modifies the interaction between electrical and mechanical parts, affecting the behavior of the entire system. The hypothesis that the inductance can be neglected has been used as a strategy to reduce the number of equations in the initial value problem. With the neglect, the system is decoupled!

To quantify how the neglect of the inductance affects the dynamics, we performed simulations neglecting the inductance and not neglecting it and comparing the two results. The results show a big difference between the two dynamics. Hence, we showed that he hypothesis that the inductance can be neglected based only on the values of the system parameters can not be made in general. Unfortunately, due to space limitation, we cannot show results for the system coupled with the crank mechanism. In it the differences are even greater.

Acknowledgments

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