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Conservation laws for a BKP-NNV type system

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In this work, we introduce the system

$$\begin{aligned} F_1 \equiv u_t + a(uv + \epsilon u_{xx})_x + b(uw + \epsilon u_{yy})_y = 0, \\ F_2 \equiv u_x + \kappa v_y = 0,\ F_3 \equiv u_y + \sigma w_x = 0, \end{aligned} \tag{1}$$

with the purpose of establishing its nonlinear self-adjointness. Determined the Lie point symmetries, we then get conservation laws via Ibragimov's Theorem.

In (1), the constants $a, b, \epsilon, \kappa, \sigma \in \mathbb{R}^*$ are arbitrary. Particularly when $\kappa = \sigma = -1$, we obtain the well-known BKP ($a = b = -6$, $\epsilon = 1/6$) and NNV ($a = b = -3$, $\epsilon = 1/3$) equations, both a symmetric generalization of the KdV equation to $(2+1)$ dimensions. Of great physical interest, especially in hydrodynamics per describing the interactive behaviors of nonlinear waves, these models, which are also known to be completely integrable, have been investigated by several authors under different approaches. For more details, see [1, 2] and references therein.

In what follows, all functions are smooth.

1 Self-Adjointness Classification

Calculated the adjoint equations,

$$\begin{aligned} F_1^* \equiv \bar{u}_t + av\bar{u}_x + bw\bar{u}_y + (\bar{v} + a\epsilon\bar{u}_{xx})_x + (\bar{w} + b\epsilon\bar{u}_{yy})_y = 0, \\ F_2^* \equiv au\bar{u}_x + \kappa\bar{v}_y = 0,\ F_3^* \equiv bu\bar{u}_y + \sigma\bar{w}_x = 0, \end{aligned}$$

where $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the new dependent variables, the nonlinear self-adjointness of the system (1) is established from the conditions

$$F_i^*|_{(\bar{u},\bar{v},\bar{w})=(\varphi,\rho,\psi)} = \lambda_i^j F_j, \quad i, j \in \{1, 2, 3\}.$$

Here λ_i^j are coefficients to be determined and

$$\varphi = \varphi(t, x, y, u, v, w), \quad \rho = \rho(t, x, y, u, v, w), \quad \psi = \psi(t, x, y, u, v, w)$$

functions such that not all vanish simultaneously. Further considerations, which will not be presented for sake of brevity, lead us to the result below.

Theorem 1. *The BKP-NNV system is nonlinearly self-adjoint with*

$$\varphi = f(t), \quad \rho = g(t)x + h(t), \quad \psi = -[f'(t) + g(t)]y + i(t).$$

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2 Conservation Laws

The components of the conserved vector $C = (C^t, C^x, C^y)$ associated to

$$X = \mathcal{T}\frac{\partial}{\partial t} + \mathcal{X}\frac{\partial}{\partial x} + \mathcal{Y}\frac{\partial}{\partial y} + \mathcal{U}\frac{\partial}{\partial u} + \mathcal{V}\frac{\partial}{\partial v} + \mathcal{W}\frac{\partial}{\partial w},$$

a infinitesimal symmetry (Lie point, Lie-Backlund, nonlocal) admitted by the system (1), are according to Ibragimov's Theorem given by

$$C^t = \varphi W^u, \quad C^x = [a\varphi(v + \epsilon D_x^2) + \rho]W^u + au\varphi W^v + \sigma\psi W^w,$$
$$C^y = [b\varphi(w + \epsilon D_y^2) + \psi]W^u + \kappa\rho W^v + bu\varphi W^w,$$

where $W^u = \mathcal{U} - \mathcal{T}u_t - \mathcal{X}u_x - \mathcal{Y}u_y$, $W^v = \mathcal{V} - \mathcal{T}v_t - \mathcal{X}v_x - \mathcal{Y}v_y$ and $W^w = \mathcal{W} - \mathcal{T}w_t - \mathcal{X}w_x - \mathcal{Y}w_y$. Once known the Lie point symmetries of the BKP-NNV system,

$$X_A = 3A(t)\frac{\partial}{\partial t} + A'(t)\left[x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} - 2\left(u\frac{\partial}{\partial u} + v\frac{\partial}{\partial v} + w\frac{\partial}{\partial w}\right)\right] + A''(t)\left(\frac{x}{a}\frac{\partial}{\partial v} + \frac{y}{b}\frac{\partial}{\partial w}\right),$$
$$X_B = B(t)\frac{\partial}{\partial x} + \frac{1}{a}B'(t)\frac{\partial}{\partial v}, \quad X_C = C(t)\frac{\partial}{\partial y} + \frac{1}{b}C'(t)\frac{\partial}{\partial w},$$

taking into account Theorem 1, the corresponding conserved vectors can be calculated.

Theorem 2. ***i)*** *From X_A, we obtain*

$$C^t = f(t)u, \quad C^x = af(t)(uv + \epsilon u_{xx}) - \sigma f'(t)yw, \quad C^y = bf(t)(uw + \epsilon u_{yy}) - f'(t)yu$$

and

$$C^t = 0, \quad C^x = f(t)(xu - \sigma yw), \quad C^y = -f(t)(yu - \kappa xv).$$

ii) *From X_A and X_B,*

$$C^t = 0, \quad C^x = f(t)u, \quad C^y = \kappa f(t)v.$$

iii) *From X_A and X_C,*

$$C^t = 0, \quad C^x = \sigma f(t)w, \quad C^y = f(t)u.$$

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