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Discrete Fourier transform and fractional equations

Eliana Contharteze Grigoletto[1]
Departamento de Bioprocessos e Biotecnologia, FCA, UNESP, Botucatu, SP
Junior Cesar Alves Soares[2]
Departamento de Matemática, UNEMAT, Barra do Bugres, MT
Edmundo Capelas de Oliveira[3]
Departamento de Matemática Aplicada, IMECC, UNICAMP, SP

In the fractional calculus [4] we have several ways to introduce the concept of fractional derivative [2]. Contharteze et al. [3] discuss a fractional version of the fundamental theorem of calculus associated with the derivative in the Caputo sense and Riemann-Liouville sense.

We use the discrete Fourier transform method [1], which is one of the most basic methods in signal analysis, to implement an algorithm to express the solution for the diffusion of neutrons in a material medium, described through a fractional differential equation [6].

In a recent paper [5], analytical and numerical methods have been proposed for solving fractional differential equations. We present a numerical approximation for the inverse Fourier transform of the function

$$\widehat{\Phi}(\omega,t)=\mathrm{E}_\alpha(-\nu\omega^\beta t^\alpha),$$

with respect to the variable ω at the point $t>0$, where $\alpha,\beta,\nu>0$, which appears in the analytical solution of many fractional differential equations, using the discrete Fourier transform. $\mathrm{E}_\alpha(\cdot)$ is the classical Mittag-Leffler function. The inverse Fourier transform of $\widehat{\Phi}(\omega,t)$ is given by

$$\Phi(x,t)=\frac{1}{\pi}\int_0^\infty \mathrm{E}_\alpha(-\nu\omega^\beta t^\alpha)\cos(\omega x)\mathrm{d}\omega. \tag{1}$$

Fixed a t_0, there exists an ω_c so that $\widehat{\Phi}(\omega_c,t_0)\cong 0$ for $|\omega|>\omega_c$.

We redefine $\widehat{\Phi}(\omega,t_0)$ as follows

$$\widehat{\Phi}(\omega,t_0)=\widehat{\Phi}(\omega,t_0),\ \text{if } 0\le\omega\le\omega_c,,\ \text{and } \widehat{\Phi}(\omega,t_0)=0,,\ \text{if } \omega>\omega_c.$$

In equation (1), if $\Delta\omega=\dfrac{\omega_c}{n}$ and $\omega_j=j\Delta\omega$, the function $\Phi(x,t_0)$ can be approximated as

$$\Phi(x,t_0)\cong\frac{1}{\pi}\int_0^{\omega_c}\widehat{\Phi}(\omega,t_0)\cos(\omega x)\mathrm{d}\omega\cong\frac{\Delta\omega}{\pi}\sum_{j=0}^{n}\widehat{\Phi}\left(\omega_j,t_0\right)\cos\left(\omega_j\,x\right). \tag{2}$$

[1] elianac@fca.unesp.br
[2] juniorcasoares@unemat.br
[3] capelas@ime.unicamp.br

We apply the numerical approximation, as described above, to obtain the solution of the so-called fractional slowing-down of neutrons [6], satisfying

$$\begin{gathered} {}^{\mathrm{C}}\mathrm{D}^{\alpha}_{0+} u(x,t) = -\nu\,(-\Delta)^{\beta/2}\,u(x,t) + \mathrm{F}(x)\delta(t), \\ u(x,0^+) = \mathrm{F}(x),\ u(x,0^-) = 0, \\ \lim_{|x|\to\infty} u_t(x,t) = 0, \end{gathered} \tag{3}$$

where $0 < \alpha \le 1,\ 1 < \beta \le 2,\ \nu > 0,\ {}^{\mathrm{C}}\mathrm{D}^{\alpha}_{0+}$ is the Caputo left-sided fractional derivative and $(-\Delta)^{\beta/2} = \mathbb{D}^{\beta}$ is the Riesz fractional derivative [4]. Particular cases for $u(x,t)$ are presented graphically in Fig. 1, below.

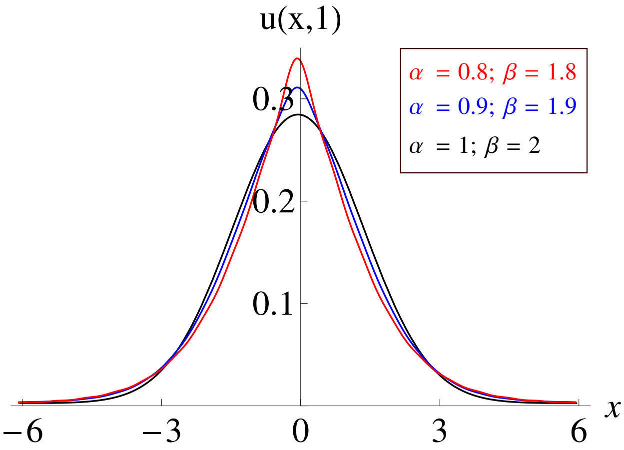

Figure 1: Graphic for $u(x,1)$ when $\mathrm{F}(x) = \delta(x)$.

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