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Computational Model of a Heart Chamber through Navier-Stokes equations

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The aim of the present work is to build a computational model of a human heart chamber through Navier-Stokes equations for the fluid, considering a bidimensional domain Ω for the cavity, whose thickness h can change with position and time.

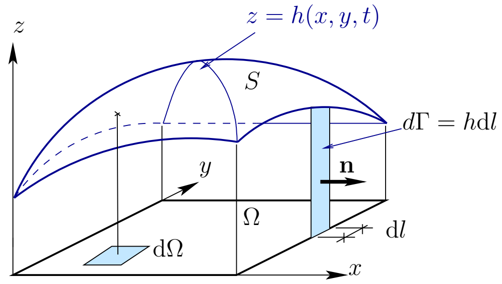

Figure 1: Thickness of the heart chamber as function of position and time t.

From a numerical perspective the finite element method was used towards solving the problem, using the mixed formulation for velocity and pressure: find the velocity $\boldsymbol{u} \in \mathcal{U}$ and pressure $p \in \mathcal{P}$ such that

$$\begin{aligned} 2\mu\Delta t \int_\Omega h\boldsymbol{\epsilon}(\boldsymbol{u}) : \boldsymbol{\epsilon}(\boldsymbol{w}) d\Omega - \Delta t \int_\Omega hp(\boldsymbol{\nabla} \cdot \boldsymbol{w}) d\Omega - 2\mu\Delta t \int_{\partial\Omega} h(\boldsymbol{\epsilon}(\boldsymbol{u})\boldsymbol{w}) \cdot \boldsymbol{n} d\Gamma \\ +\rho \int_\Omega h\boldsymbol{u} \cdot \boldsymbol{w} d\Omega + \Delta t \int_\Omega (\boldsymbol{\nabla}\boldsymbol{u})\boldsymbol{u} \cdot \boldsymbol{w} d\Omega = \Delta t \int_\Omega h\boldsymbol{f} \cdot \boldsymbol{w} d\Omega \\ +\rho \int_\Omega h\boldsymbol{u} \cdot \boldsymbol{w} d\Omega - \Delta t \int_{\partial\Omega} hp\boldsymbol{w} \cdot n d\Gamma \qquad \forall \boldsymbol{w} \in \mathcal{U} \quad (1) \end{aligned}$$

$$-\int_\Omega h\boldsymbol{\nabla} \cdot \boldsymbol{u} q d\Omega - \int_\Omega \boldsymbol{u} \cdot \boldsymbol{\nabla} h d\Omega = 0 \qquad \forall q \in \mathcal{P}. \quad (2)$$

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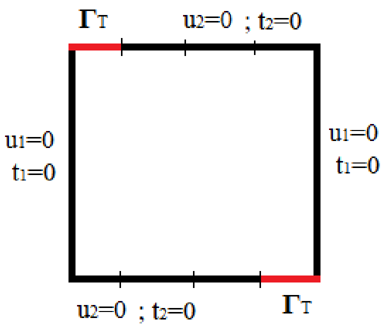
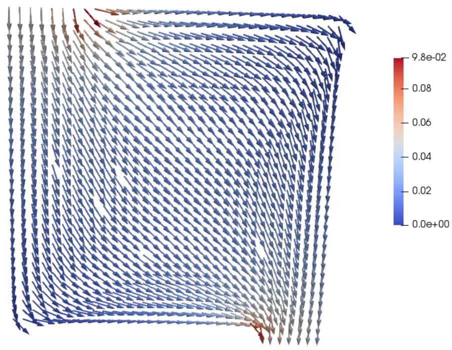
Where $\mathcal{U} = H^2(\Omega, \mathbb{R}^2)$, $\mathcal{P} = L^2(\Omega, \mathbb{R})$, ρ and μ stands for the fluid density and viscosity, respectively. The term $\boldsymbol{f}$ indicates the body forces applied on the fluid.

Between several possible choices [3], the chosen inf-sup stable subspaces $\mathcal{U}_h := \{\boldsymbol{u}_h \in \mathcal{U}; \quad \boldsymbol{u}_h|_E \in \boldsymbol{Q}_E(\mathbb{Q}_k(\hat{E}, \mathbb{R}^2)) \quad \forall E \in \{E\}\}$ defined through a bilinear mapping $\boldsymbol{Q}_E$ and $\mathcal{P}_h := \{p_h \in \mathcal{P}; \quad p_h|_E \in \mathbb{P}_{k-1}(E, \mathbb{R}^2) \quad \forall E \in \tau_h\}$ defined over the geometric element. The approximation in time is done using the Euler Implicit method associated with a sequential algorithm in order to solve the non-linearity in (1).

Simulations of blood flow rate inside the chamber were carried out with a program written in Fortran language. Figure (2) represents the geometry and the velocity field. It exhibits the vector field related to the velocity pattern in a fixed time and illustrates the approximated geometry for the chamber together with the chosen boundary conditions so that $\boldsymbol{t} = (t_1, t_2)$ denotes the traction vector and Γ_T assigns the traction condition, in which the pressure is weakly prescribed.



(a) Domain and boundary conditions



(b) Velocity Field profile with bottom valve opened

Figure 2: Chamber domain and velocity field.

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