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On time reversibility in stochastic differential equations

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1 Introduction

Stochastic differential equations and its applications are a subject of great interest for scientific research. In this area, Langevin and Fokker-Planck formalisms are extensively used. Systems modeled by differential stochastic equations with additive noise have been largely studied and are the most popular models. However, the understanding of the stochastic dynamics and the evolution to equilibrium for systems dealing with multiplicative noise is difficult and there is a lack of general tools for its characterization. In particular, for the multiplicative noise case, the Fokker-Planck equation does depend on the chosen prescription for the stochastic integration of the associated Langevin equation.

In this work [2,4,5], we study equilibrium properties of Markovian multiplicative white-noise processes. For this, we carefully define the time reversal transformation for this kind of processes, taking into account that the asymptotic stationary probability distribution depends on the prescription. In white noise multiplicative processes, stochastic trajectories evolve with different prescriptions in one direction and in the reverse direction. We show that, using a careful definition of equilibrium distribution and taking into account the appropriate time reversal transformation, usual equilibrium properties are satisfied for any prescription.

2 The model

We consider a single random variable $x(t)$ satisfying a first order differential equation (Langevin equation) given by

$$\frac{dx(t)}{dt} = f(x(t)) + g(x(t))\eta(t), \tag{1}$$

where $\eta(t)$ is a Gaussian white noise, $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \delta(t - t')$. The drift force $f(x)$ and the diffusion function $g(x)$ are, in principle, arbitrary smooth functions of $x(t)$.

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This kind of equations is commonly used to describe many diffusion processes occurring in nature under some random influence.

To correctly define Eq. (1), it must be specified a prescription for performing the stochastic integration. We consider a general prescription α, where α is defined as a continuous parameter, $0 \leq \alpha \leq 1$. Each of its values corresponds with a different discretization rule for the stochastic differential equation. $\alpha = 0$ corresponds with Itô prescription at the time that $\alpha = 1/2$ corresponds with Stratonovich one. We also represent the stochastic process in a functional Grassmann formalism [1,3,4], which turns out to be very convenient for handling stochastic trajectories.

3 Conclusions

We found an asymptotic equilibrium distribution by solving the stationary Fokker-Planck equation. This conduces to an equilibrium potential $U_{eq}(x)$ which explicitly depends on α. Regarding time reversibility, we shown that, if the forward stochastic trajectory evolves with a definite value of α, the time reversed trajectory evolves with the conjugated prescription $(1-\alpha)$. Therefore, in order to have a unique equilibrium distribution, the definition of time reversed stochastic process is given by a specific transformation, where we need to change not only the velocity sign, but also the prescription and the drift force.

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